Ralph [illegible]

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION
GROUND WATER BRANCH
Washington 25, D. C.

GROUND WATER NOTES
HYDRAULICS

No. 7

January 1953

A GENERALIZED GRAPHICAL METHOD OF EVALUATING FORMATION CONSTANTS AND SUMMARIZING WELL-FIELD HISTORY

By
H. H. Cooper, Jr. and C. E. Jacob

This paper was originally published in the Transactions of the American Geophysical Union, volume 27, number 4, August 1946, pp. 526-534. Reprints are not available, so it is here presented as number 7 of the series of Ground Water Notes for the use of professional personnel of the Ground Water Branch. With the exception of the correction of typographical errors and the addition of a note at the end of the paper, no changes in the original paper have been made.

Abstract--The capacities of a water-bearing formation to transmit water under a hydraulic gradient and to yield water from storage when the water table or artesian pressure declines, are generally expressed, respectively, in terms of a coefficient of transmissibility and a coefficient of storage. Determinations of these two constants are almost always involved in quantitative studies of ground-water problems.

C. V. Theis (1935) gave an equation, adapted from the solution of the analogous problem in heat conduction, for computing the non-steady drawdown accompanying the radial flow of water to a well of constant discharge. This equation has been used successfully many times for determining coefficients of transmissibility and storage from observed drawdowns. As it involves a transcendental function known as the exponential integral and two unknown coefficients, one of which occurs both in the argument and as a divisor of the function, the coefficients cannot be determined directly. However, they may be determined by a graphical method devised by Theis and described by Jacob (1940, p. 582) and Wenzel (1942, pp. 88-89). This method requires the use of a "type curve", on which the observed data are superimposed to determine the coefficients.

Later, Wenzel and Greenlee (1944) gave a generalization of Theis' graphical method by which the coefficients may be determined from tests of one or more discharging wells operated at changing rates. This method requires the computation of a special type curve for each observation of drawdown used. It is without doubt a worth-while contribution to the quantitative techniques of ground-water hydraulics, but in tests that involve more than a very few discharging wells or a very few changes in the rates of discharge, the computation of the special type curves is necessarily so laborious as to make the method difficult to apply.

The present paper gives a simple straight-line graphical method for accomplishing the same purposes as the methods developed by Theis and Wenzel and Greenlee. Type curves are not required. The writers believe that the straight-line method, where applicable, has decided advantages, in ease of application and interpretation, over the other graphical methods. However, as the method will not be applicable in some cases, it is expected to supplement, rather than supersede, the other methods. The method is designed especially for artesian conditions, but it may be applied successfully to tests of non-artesian acuifers under favorable circumstances.

This paper first gives the development of the method for tests involving a single discharging well operating at a steady rate, and then generalizes the method to make it applicable to tests involving one or more wells discharging intermittently or at changing rates. Examples are given to demonstrate the method.

Straight-line method for a single well discharging at a steady rate

When sufficient time has elapsed after an artesian well has begun discharging at a steady rate, the drawdown within a given distance increases approximately in proportion to the logarithm of the time since the discharge began, and decreases in proportion to the logarithm of the distance from the well. By virtue of this relationship, it is possible to determine the coefficients of transmissibility and storage of an acuifer from a simple semi-logarithmic plot of observed drawdowns.

The drawdown produced by a well discharging at a steady rate from an extensive artesian aquifer of uniform thickness and permeability is given by equation (1) (Theis, 1935).

$$s = (Q/4\pi T)W(u)$$

$$= (Q/4\pi T)(-0.5772 - \log_e u + u - u^2/2.2! + u^3/3.3! - \ldots) \qquad (1)$$

Here $u = r^2S/4Tt$; r = distance from the discharging well, t = time elapsed since start of discharge, T = transmissibility of the aquifer (discharge per unit normal width per unit hydraulic gradient), S = coefficient of storage (volume of water that a unit decline of head releases from storage in a vertical prism of the aquifer of unit cross section), and Q = discharge of the well.

For small values of (r^2/t) compared to the value of $(4T/S)$, u will be so small that the series following the first two terms in the series in equation (1) may be neglected. Thus, where values of (r^2/t) are relatively small, equation (1) may, for all practical purposes, be approximated as in equation (2).

$$s = (Q/4\pi T)\left[\log_e(1/u) - 0.5772\right]$$

$$= (Q/4\pi T)\left[\log_e(4Tt/r^2S) - 0.5772\right]$$

or $$s = (Q/4\pi T)\log_e(4e^{-0.5772}Tt/r^2S) = (Q/4\pi T)\log_e(2.25Tt/r^2S) \qquad (2)$$

The approximation will be tolerable where u is less than about 0.02. Converting to the common logarithm, we may rewrite equation (2) in any one of the three forms in equations (3), (4), and (5).

$$s = -(2.303Q/2\pi T)\left[\log_{10}r - (1/2)\log_{10}(2.25Tt/S)\right] \qquad (3)$$

$$s = (2.303Q/4\pi T)\left[\log_{10}t - \log_{10}(r^2S/2.25T)\right] \qquad (4)$$

$$s = -(2.303Q/4\pi T)\left[\log_{10}(r^2/t) - \log_{10}(2.25T/S)\right] \qquad (5)$$

The only variables in these equations are the drawdown s, the distance r, and the time t. It is apparent than when t is constant, (3) will be the equation of the straight-line plot of s against $\log_{10}r$. Similarly, when r is constant, (4) will be the equation of the straight-line plot of s against $\log_{10}t$. Moreover, with r and t combined into the single variable (r^2/t), (5) will be the equation of the straight-line plot of s against $\log_{10}(r^2/t)$.

In each equation the slope of the corresponding straight-line plot is represented by the quantity on the outside of the brackets, and the intercept of the straight line on the zero-drawdown line is represented by the second term within the brackets.

As T is the only unknown in the quantity representing the slope, the coefficient of transmissibility is readily determined from a semi-logarithmic plot of observed data by equating the slope of the plot with the corresponding quantity in equation (3), (4), or (5), and solving for T. After T is determined, the only unknown remaining in the term representing the intercept will be S. Therefore, the coefficient of storage may then be determined by equating the intercept of the plot with the corresponding term, and solving for S.

The plots will be straight lines only where (r^2/t) is relatively small so that u is small. A measurement of drawdown that is made too soon after the discharge is begun, or too far from the discharging well, will plot not on the straight line, but on a curve asymptotic to it. However, in tests of artesian aquifers u becomes small soon after the discharge is begun, and hence in most cases little, if any, of the data will fall off the straight line.

The three types of graphs that correspond respectively to equations (3), (4), and (5) may be referred to as the distance-drawdown graph, the time-drawdown graph, and the composite-drawdown graph. The type of graph to be selected for determining the coefficients from a given discharging-well test will depend on the set of data collected in the field.

Distance-drawdown graph--This is a graph of the drawdown at a time t after the discharge begins, plotted against r on semi-logarithmic paper with r on the logarithmic scale. It may be thought of as a radial profile of the (logarithmic) cone of depression. Equating the quantity outside of the brackets in equation (3) with the slope of the graph, $2.303Q/2\pi T = \Delta s/\Delta \log_{10} r$ = slope of plot, whence $T = -(2.303Q/2\pi)(\Delta \log_{10} r/\Delta s)$. The negative sign indicates that s decreases as $\log_{10} r$ increases. For convenience, $\Delta \log_{10} r$ may be made unity by having it represent one logarithmic cycle, whereupon

$$T = -2.303Q/2\pi\Delta s \qquad (6)$$

where Δs is the difference in drawdown over one logarithmic cycle.

Equating the second term in brackets in equation (3) with the intercept of the straight line on the zero-drawdown line, and solving for the coefficient of storage, gives equation (7).

$$S = 2.25Tt/r_0^2 \qquad (7)$$

where r_0 is the value of r at the s = 0-intercept.

Figure 1 is a distance-drawdown graph for wells that are 49, 100, and 150 feet from another well discharging at the rate of 2.23 cfs (test by S. W. Lohman reported by Wenzel, 1942). The drawdowns at these distances after 18 days of continuous discharge were 5.09, 4.08, and 3.10 feet, respectively. The difference in drawdown over one logarithmic cycle is (0.69 ft - 4.07 ft) = -3.38 ft. Therefore, from equation (6), T = 2.303 (2.23 cfs)/(2 x 3.38 ft) = 0.242 cfs/ft.

The straight line drawn through the plotted points intersects the zero-drawdown line at r_0 = 1600 ft. Thus, from equation (7), S = 2.25(0.242 cfs/ft)(18 days x 86,400 sec/day)/(1600 ft)^2 = 0.33.

Time-drawdown graph--This graph is a plot of the drawdowns in one of the observed wells against t on semi-logarithmic paper, with t on the logarithmic scale. The formulas for T and S are as in equations (8) and (9).

$$T = 2.303Q/4\pi\Delta s \qquad (8)$$

$$S = 2.25Tt_0/r^2 \qquad (9)$$

where t_0 is the value of t at the intercept.

Figure 2 is a time-drawdown graph for a well 1200 feet from another well discharging 3.00 cfs from a confined aquifer (Jacob, 1946).

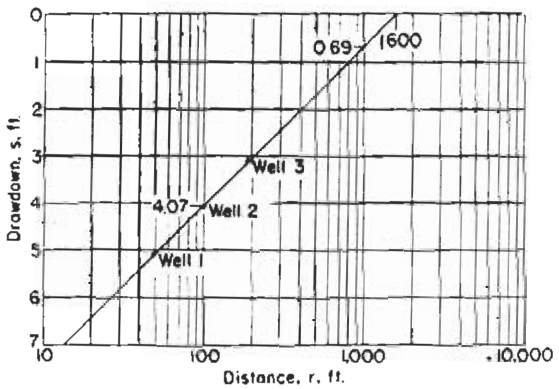

Fig. 1--Distance-drawdown graph based on drawdowns in three wells after 18 days of continuous discharge from an unconfined sand, Q = 2.23 cfs

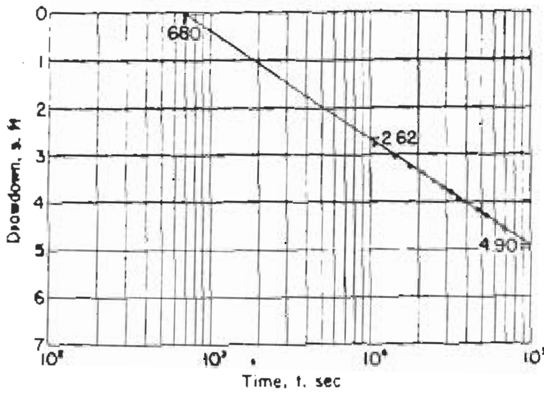

Fig. 2--Time-drawdown graph for a wel' 1,200 feet from another well discharging from a confined sand, Q = 3.00 cfs

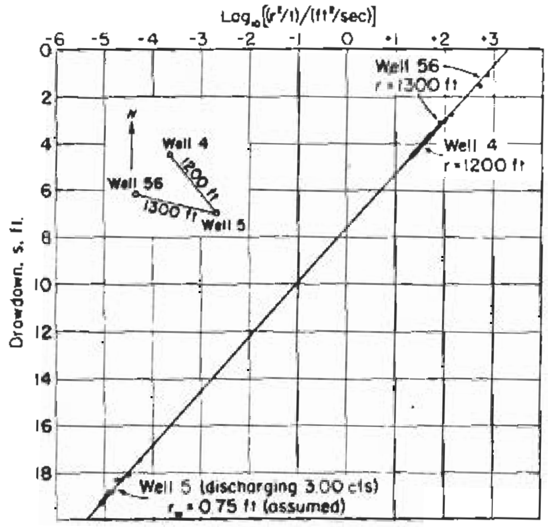

Fig. 3--Composite drawdown graph based on drawdowns observed in a discharging well and two neighboring wells in a confined sand (compare with Fig. 2)

The plotted points represent water-level readings from an automatic water-stage recording instrument, selected first at one-hour intervals and later at two-hour intervals. The change in drawdown over one logarithmic cycle is 2.28 feet. Accordingly, from equation (8), T = 2.303 (3.00 cfs)/(4π x 2.28 ft) = 0.241 cfs/ft.

The fact that this value for the coefficient of transmissibility agrees closely with that in the preceding example is fortuitous inasmuch as the two sets of data are from tests on different aquifers.

The intercept on the zero-drawdown line is t_0 = 680 seconds. Therefore, from equation (9), S = 2.25(0.241 cfs/ft)(680 sec)/(1200 ft)^2 = 0.00026.

<u>Composite drawdown graph</u>--This graph is a plot of the drawdowns in several observed wells at different times against (r^2/t), on semi-logarithmic paper. The formulas for the coefficients of transmissibility and storage are as in equations (10) and (11).

$$T = -(2.303Q/4\pi)/\Delta s \tag{10}$$

$$S = 2.25T/(r^2/t)_0 \tag{11}$$

where $(r^2/t)_0$ is the value of (r^2/t) at the intercept.

Figure 3 is a composite drawdown graph that includes, in addition to the drawdowns in Figure 2, the drawdowns in a second idle well 1300 feet from the discharging well, and the drawdowns in the discharging well itself. The drawdowns in the discharging well are adjusted for an inferred screen loss of 28.5 feet (Jacob, 1946). The discharging well is gravel-walled and its screen has a nominal diameter of 18 inches. The effective radius of the well is assumed to be 0.75 foot.

The change in drawdown over one logarithmic cycle is -2.31 feet. This value substituted in equation (10) gives a coefficient of transmissibility of 0.238 cfs/ft. Inasmuch as the measurement of the discharge is correct only to two significant figures, this value does not differ significantly from that determined from Figure 2.

The intercept on the zero-drawdown line is $(r^2/t)_0$ = 2000 sq ft/sec. From this value, the coefficient of storage is computed to be 0.00027, which agrees closely with the value determined from Figure 2.

Generalized straight-line method

Before proceeding with the generalization of the straight-line method, it will be necessary to adopt a set of distinctive symbols to represent the various physical elements involved. The numerals 1, 2, 3, ... will be used to identify the observed wells, and the letter i will be the general symbol for indicating any one of them. Thus, "Well i" will be understood to mean Well 1, Well 2, Well 3, etc., in turn. Other symbols are: ΔQ_k = increment of discharge for k = 1, 2, 3, 4, ... n; t^k = time elapsed since the inception of ΔQ_k for $t^k = t', t'', t''', t^{iv}, \ldots t^n$; r_{ik} = distance from observed well i to the discharging well in which ΔQ_k occurred; $\Delta s_i{}^k$ = partial drawdown in observed well i produced by the increment of discharge ΔQ_k at the time t^k;

$$Q_n = \Delta Q_1 + \Delta Q_2 + \Delta Q_3 + \ldots \Delta Q_n = \sum_{k=1}^{n} \Delta Q_k$$

which is the algebraic sum of increments of discharge ΔQ_1 to ΔQ_n; and $s_i{}^n$ = total drawdown in observed well i produced by increments of discharge ΔQ_1 to ΔQ_n.

An increment of discharge ΔQ_k may be the initial discharge or a subsequent increase or decrease in discharge in any one of the discharging wells. Increases in discharge will be positive increments, and decreases will be negative. It will be convenient to assign numerals to k in chronological order, but where two or more increments of discharge occur simultaneously, the numerals may be assigned arbitrarily.

In the treatment of problems involving multiple discharging wells, or changes in the discharge of a single well, use is made of the principle of superposition, whereby it is assumed that the total drawdown produced in a given well at a given time by several increments of discharge is the algebraic sum of the drawdowns that would be produced independently by those increments of discharge. So far, the results of discharging-well tests have verified this assumption for artesian conditions.

Equation (12) is according to the principle of superposition.

$$s_i{}^n = \Delta s_i{}' + \Delta s_i{}'' + \Delta s_i{}''' + \ldots \Delta s_i{}^n = \sum_{k=1}^{n} \Delta s_i{}^k \qquad (12)$$

From equation (2) the partial drawdown produced in an observed well i by an increment of discharge ΔQ_k is approximately $\Delta s_i{}^k = (2.303\Delta Q_k/4\pi T)\log_{10}(2.25Tt^k/r^2{}_{ik}S)$, and from equation (12) the total drawdown, after n increments of discharge is in equation (13), for n = 1, 2, 3, etc.

$$s_i{}^n = \sum_{k=1}^{n} \Delta s_i{}^k = \sum_{k=1}^{n}\left[\frac{2.303\Delta Q_k}{4\pi T}\log_{10}\frac{2.25Tt^k}{r^2{}_{ik}S}\right] \qquad (13)$$

Dividing both sides of equation (13) by Q_n, equation (13a) results

$$s_i{}^n/Q_n = \sum_{k=1}^{n}\left[\frac{2.303\Delta Q_k}{4\pi TQ_n}\log_{10}\frac{2.25Tt^k}{r^2{}_{ik}S}\right] \qquad (13a)$$

This may be written as in equation (14) or (15)

$$\left(\frac{s}{Q}\right)_i^n = -\frac{2.30}{4\pi T}\left[2\sum_{k=1}^{n}\frac{\Delta Q_k}{Q_n}\log_{10}r_{ik} - \sum_{k=1}^{n}\frac{\Delta Q_k}{Q_n}\log_{10}t^k - \log_{10}\frac{2.25T}{S}\right] \qquad (14)$$

$$\left(\frac{s}{Q}\right)_i^n = -\frac{2.30}{4\pi T}\left[\sum_{k=1}^{n}\frac{\Delta Q_k}{Q_n}\log_{10}\frac{r^2}{t}{}_i{}^k - \log_{10}\frac{2.25T}{S}\right] \qquad (15)$$

The first and second terms in brackets in equation (14) and the first term in brackets in equation (15) are the logarithms of the weighted logarithmic means of r^2, t, and (r^2/t) respectively. The weighted logarithmic means may be represented by $\bar{r}_{in}$, $\bar{t}^n$, and $\overline{(r^2/t)}_i^{\,n}$. Substituting these symbols in equations (14) and (15), we may now write the three equations (16), (17), and (18).

$$\left(\frac{s}{Q}\right)_i^n = -\frac{2.303}{2\pi T}\left[\log_{10}\bar{r}_{in} - \frac{1}{2}\log_{10}\frac{2.25T\bar{t}^n}{S}\right] \qquad (16)$$

$$\left(\frac{s}{Q}\right)_i^n = \frac{2.303}{4\pi T}\left[\log_{10}\bar{t}^n - \log_{10}\frac{\bar{r}^2{}_{in}S}{2.25T}\right] \qquad (17)$$

$$\left(\frac{s}{Q}\right)_i^n = -\frac{2.303}{4\pi T}\left[\log_{10}\overline{(r^2/t)}_i^{\,n} - \log_{10}\frac{2.25T}{S}\right] \qquad (18)$$

These equations correspond with equations (3), (4), and (5) for single discharging wells, but include in addition to $\bar{s}_i^n$, $\bar{r}_{in}$, and $\bar{t}^n$, a fourth variable, Q_n. So that equations (16), (17), and (18) will be the equations of straight-line plots, Q_n has been combined with s_i^n into a single variable $(s/Q)_i^n$, which may be referred to as the "specific drawdown" (drawdown per unit discharge). Thus, (16), (17), and (18) are the equations of the straight-line plots of the specific drawdown against the logarithms of $\bar{r}_{in}$, $\bar{t}^n$, and $\overline{(r^2/t)}_i^{\,n}$, respectively, where $\bar{t}^n$ is constant in equation (16), $\bar{r}_{in}$ is constant in equation (17), and $\bar{r}_{in}$ and $\bar{t}^n$ are combined into a single variable in equation (18). As in equations (3), (4), and (5), the slope of each plot is represented by the quantity on the outside of the brackets in the corresponding equation, and the intercept of the extension of the plot at $(s/Q)_i^n = 0$ is represented by the second term within the brackets.

The weighted logarithmic mean distance $\bar{r}_{in}$ for a given observed well at a given time may be computed in the following manner: (1) Multiply each increment of discharge that occurred before the given time by the logarithm of the distance from the observed well to the well in which the increment occurred; (2) sum the products algebraically; (3) divide the sum of the products by the algebraic sum of the increments of discharge; and (4) extract the antilogarithm of the quotient. The result will be the distance $\bar{r}_{in}$. The weighted logarithmic means $\bar{t}^n$ and $\overline{(r^2/t)}_i^{\,n}$ are computed in a similar manner, but where $\bar{r}_{in}$ and $\bar{t}^n$ are already computed, $\overline{(r^2/t)}_i^{\,n}$ may be obtained more conveniently by dividing $\bar{r}^2{}_{in}$ by $\bar{t}^n$ directly.

The weighted logarithmic means $\bar{r}_{in}$ and $\bar{t}^n$ both have physical significance. From a comparison of equation (16) with equation (3) it is evident that $\bar{r}_{in}$ is the distance at which a single well discharging at a rate Q_n would produce the drawdown s_i^n at the elapsed time $\bar{t}^n$ after the discharge began. A recognition of the significance of these quantities is helpful in interpreting the plots.

The three types of graphs corresponding, respectively, to equations (16), (17), and (18) are referred to as the <u>generalized distance-drawdown graph</u>, the <u>generalized time-drawdown graph</u>, and the <u>generalized composite drawdown graph</u>. The formulas for determining the coefficients of transmissibility and storage from these graphs may be derived in the same manner as in the method for a single well discharging uniformly; that is, by equating the slopes and the intercepts of the plots with the corresponding quantities in the respective equations. The formulas are as in the following paragraphs.

Generalized distance-drawdown graph

$$T = \frac{-2.303}{2\pi\Delta\left(\frac{s}{Q}\right)_i^n} \qquad (19)$$

where $\Delta \frac{s}{Q}{}_i^n$ is the change in specific drawdown over one logarithmic cycle.

$$S = \frac{2.25T\bar{t}_n}{\bar{r}_0^2} \qquad (20)$$

where $\bar{r}_0$ is the value of $\bar{r}_{in}$ at the intercept.

Generalized time-drawdown graph

$$T = \frac{2.303}{4\pi\Delta\left(\frac{s}{Q}\right)_i^n} \qquad (21)$$

$$S = \frac{2.25T\bar{t}_0}{\bar{r}^2_{in}} \qquad (22)$$

where $\bar{t}_0$ is the value of $\bar{t}^n$ at the intercept.

Generalized composite drawdown graph

$$T = \frac{-2.303}{4\pi\Delta(s/Q)_i^n} \qquad (23)$$

$$S = \frac{2.25T}{(r^2/t)_0} \qquad (24)$$

where $(\overline{r^2/t})_0$ is the value of $(\overline{r^2/t})_i^n$ at the intercept. The use of the generalized composite drawdown graph is demonstrated in the example that follows.

Figure 4(a) shows the locations of wells at the Central Plant of the municipal water supply of Houston, Texas (Guyton and Rose, 1945). The columnar sections, based on well logs, show by stippling the sands penetrated by the wells. The positions of the well screens are also indicated.

Figure 4(b) is a graph of the drawdown and subsequent partial recovery observed in Well F5 on October 10, 1939 (Jacob, 1941). Well F10, 850 feet from Well F5, began pumping 2.27 cfs at 10^h00^m and stopped pumping at 18^h45^m. Well F1, 780 feet away, began pumping 2.79 cfs at 10^h30^m and stopped pumping at 20^h05^m. Well F12, 1060 feet away, began pumping 3.56 cfs at 11^h00^m and continued pumping through the end of the test. Measurements of the water level in Well F5 were made throughout the day. Some of these measurements, expressed as drawdowns, are plotted in Figure 4(b), where the measurements used in applying the generalized straight-line graphical method are plotted each as two concentric circles.

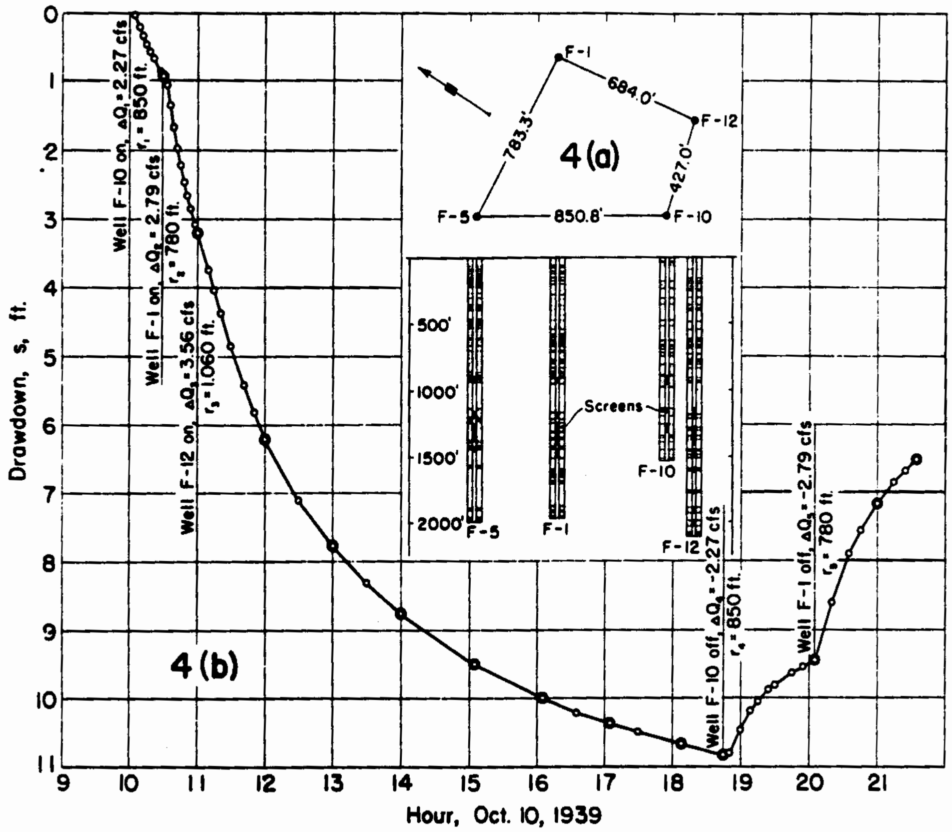

Fig. 4--(a) Relative location of wells at Central Plant, Houston, Tex., and columnar sections based on well logs (after Guyton and Rose)

(b) Drawdown and subsequent partial recovery observed in well F5, October 10, 1939, resulting from staggered operation of wells F10, F1, and F12

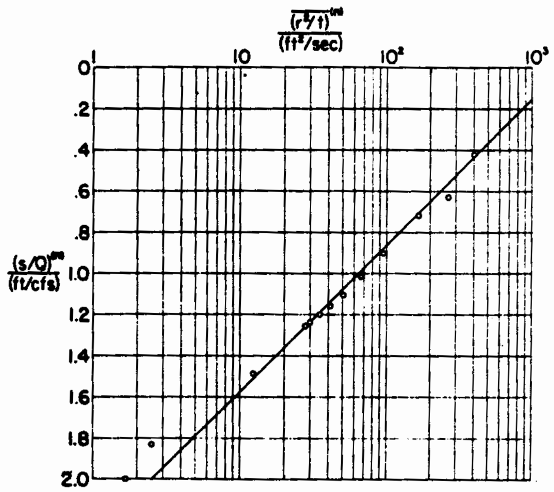

Fig. 5--Generalized composite drawdown graph for Well F5, Central Plant, Houston, Texas, October 10, 1939

Table 1--Computations of specific drawdown and weighted logarithmic mean $\overline{(r^2/t)}^n$ for Well F5, Central Plant, Houston, Texas, October 10, 1939

Time	k	n	Discharge well	r_k	t^k	(r^2_k/t^k)	$Log_{10}(r^2_k/t^k)$	ΔQ_k	(9)×(8)	$Log_{10}\,\overline{(r^2/t)}^n$	$\overline{(r^2/t)}^n$	s^n	$(s/Q)^n$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
h m				ft	sec	ft²/sec	ft²/sec	cfs	cfs		ft²/sec	ft	ft/cfs
10 30	1	1	F10	850	1800	402	2.604	2.27	....	2.604	402	0.96	0.423
11 00	1	..	F10	850	3600	201	2.303	2.27	5.23	....	....	...	....
	2	..	F1	780	1800	338	2.529	2.79	7.08	....	....	...	....
		2	...	....	...	...	....	5.08	12.29	2.429	289	3.20	0.832
12 00	1	..	F10	850	7200	100.4	2.002	2.27	4.54	....	....	...	....
	2	..	F1	780	5400	112.8	2.052	2.79	5.73	....	....	...	....
	3	..	F12	1060	3600	312	2.494	3.56	8.88	....	....	...	....
		3	...	...	...	...	....	8.62	19.15	2.222	167	6.21	0.720
13 00	1	..	F10	850	10800	86.9	1.826	2.27	4.15	....	....	...	....
	2	..	F1	780	9000	87.8	1.830	2.79	5.11	....	....	...	....
	3	..	F12	1060	7200	156	2.194	3.56	7.81	....	....	...	....
		3	...	...	...	...	....	8.62	17.07	1.980	95.5	7.77	0.901
14 00	1	..	F10	850	14400	50.2	1.701	2.27	3.86	....	....	...	....
	2	..	F1	780	12600	48.3	1.684	2.79	4.70	....	....	...	....
	3	..	F12	1060	10800	104	2.017	3.56	7.18	....	....	...	....
		3	...	...	...	...	....	8.62	15.74	1.826	67.0	8.76	1.016
15 05	1	..	F10	850	18300	39.5	1.597	2.27	3.63	....	....	...	....
	2	..	F1	760	16500	36.9	1.567	2.79	4.37	....	....	...	....
	3	..	F12	1060	14700	76.4	1.883	3.56	6.70	....	....	...	....
		3	...	...	...	...	....	8.62	14.70	1.705	50.7	9.50	1.102
16 05	1	..	F10	850	21900	33.0	1.518	2.27	3.45	....	....	...	....
	2	..	F1	760	20100	30.3	1.481	2.79	4.13	....	....	...	....
	3	..	F12	1060	18300	61.4	1.788	3.56	6.37	....	....	...	....
		3	...	...	...	...	....	8.62	13.95	1.618	41.5	10.00	1.160
17 05	1	..	F10	850	25500	28.3	1.453	2.27	3.30	....	....	...	....
	2	..	F1	780	23700	25.7	1.410	2.79	3.93	....	....	...	....
	3	..	F12	1060	21900	51.3	1.710	3.56	6.09	....	....	...	....
		3	...	...	...	...	....	8.62	13.32	1.545	35.1	10.37	1.203
18 08	1	..	F10	850	29280	24.7	1.392	2.27	3.160	....	....	...	....
	2	..	F1	780	27480	22.1	1.345	2.79	3.753	....	....	...	....
	3	..	F12	1060	25680	43.8	1.641	3.56	5.842	....	....	...	....
		3	...	...	...	...	....	6.62	12.755	1.4797	30.18	10.67	1.238
18 45	1	..	F10	850	31500	22.9	1.361	2.27	3.089	....	....	...	....
	2	..	F1	780	29700	20.5	1.311	2.79	3.658	....	....	...	....
	3	..	F12	1060	27900	40.3	1.605	3.56	5.714	....	....	...	....
		3	...	...	...	...	....	8.62	12.461	1.4456	27.90	10.84	1.258
20 05	1	..	F10	850	36300	19.9	1.299	2.27	2.949	....	....	...	....
	2	..	F1	780	34500	17.6	1.246	2.79	3.476	....	....	...	....
	3	..	F12	1060	32700	34.4	1.536	3.56	5.468	....	....	...	....
	4	..	F10	850	4800	150.5	2.177	-2.27	- 4.942	....	....	...	....
		4	...	...	...	...	....	6.35	6.951	1.0946	12.43	9.45	1.488
21 00	1	..	F10	850	39600	18.2	1.261	2.27	2.862	....	....	...	....
	2	..	F1	780	37800	16.1	1.207	2.79	3.368	....	....	...	....
	3	..	F12	1060	36000	31.2	1.494	3.56	5.319	....	....	...	....
	4	..	F10	850	8100	89.2	1.950	-2.27	- 4.427	....	....	...	....
	5	..	F1	780	3300	184.4	2.266	-2.79	- 6.322	....	....	...	....
		5	...	...	...	...	....	3.56	0.800	0.2247	1.678	7.16	2.011
21 35	1	..	F10	850	41700	17.3	1.239	2.27	2.813	....	....	...	....
	2	..	F1	780	39900	15.2	1.183	2.79	3.301	....	....	...	....
	3	..	F12	1060	38100	29.5	1.470	3.56	5.233	....	....	...	....
	4	..	F10	850	10200	70.8	1.850	-2.27	- 4.199	....	....	...	....
	5	..	F1	760	5400	112.7	2.052	-2.79	- 5.725	....	....	...	....
		5	...	...	...	...	....	3.56	1.423	0.3997	2.51	6.51	1.829

Note: The subscript i, which refers to the observation well, is omitted, because only one observation well is involved in the example.

Computations to determine values of the weighted logarithmic mean $(\overline{r^2/t})^n$ and the corresponding values of the specific drawdown $(s/Q)^n$ are given in Table 1. (The subscript i, which refers to the observation well, is omitted from the symbols because only one observation well is involved in the example.) The computation procedure may be observed by following the headings of the columns in the Table. The increments of discharge that occurred before the time given in column (1) are listed and summed algebraically in column (9). These increments of discharge are multiplied by the logarithms of the corresponding values of (r^2/t), and the products are listed and summed algebraically in column (10). The sum of the products given in column (10) is then divided by the sum of the increments of discharge given in column (9), and the quotient is listed in column (11). The antilogarithm of this quotient, listed in column (12) is the weighted logarithmic mean $(\overline{r^2/t})^n$. The corresponding value of the specific drawdown $(s/Q)^n$ is listed in column (14).

The data given in columns (12) and (14) are plotted in Figure 5. The alignment of the plotted points is not bad in view of the fact that the screens of the four wells are set at various depths and also the fact that the water-bearing sands are lenticular and vary in thickness and permeability from one well to another. The extent to which these or other circumstances might vitiate the method used may be judged most readily from the alignment of the points on a simple, straight-line graph such as Figure 5.

The change in specific drawdown $\Delta(s/Q)^n$ over one logarithmic cycle is -0.71 ft per cfs. Therefore, from equation (23) $T = 2.303/(4\pi \times 0.71 \text{ ft/cfs}) = 0.26$ cfs/ft.

The extension of the straight line in Figure 5 intersects the line of zero drawdown at $(\overline{r^2/t})^n = (\overline{r^2/t})_0{}^n = 1650$ ft²/sec. Thus, from equation (24) $S = 2.25(0.26 \text{ cfs/ft})/(1650 \text{ ft}^2/\text{sec}) = 0.00035$.

Note: Formulas given in this paper are applicable for any set of consistent units of length and time. It has been the custom of some writers and investigators to express r in feet, t in days, Q in gallons per minute, and T in gallons per day per foot, S being a dimensionless fraction. For these inconsistent units, the right-hand members of the formulas for transmissibility must be multiplied by the factor 1,440 and the right-hand members of the formulas for the coefficient of storage by the factor 0.1337.

REFERENCES

Guyton, W. F., and Rose, N. A., 1945, Quantitative studies of some artesian aquifers in Texas: Econ. Geology, vol. 40, no. 3, pp. 193-226.

Jacob, C. E., 1940, On the flow of water in an elastic artesian aquifer: Am. Geophys. Union Trans., 1940, pt. 2, pp. 574-586.

Jacob, C. E., 1941, Coefficients of storage and transmissibility obtained from pumping tests in the Houston district, Tex.: Am. Geophys. Union Trans., 1941, pt. 3, pp. 744-756.

Jacob, C. E., 1946, Drawdown test to determine effective radius of an artesian well: Am. Soc. Civil Eng. Proc., vol. 72, no. 5, pp. 629-646.

Theis, C. V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., 1935, pt. 2, pp. 519-524.

Wenzel, L. K., 1942, Methods for determining permeability of water-bearing materials: U. S. Geol. Survey Water-Supply Paper 887.

Wenzel, L. K., and Greenlee, A. L., 1944, A method for determining transmissibility and storage coefficients by tests of multiple well systems: Am. Geophys. Union Trans., 1943, pt. 2, pp. 547-560.

34973

REFERENCES

Guyton, W. F., and Rose, N. A., 1945, Quantitative studies of some artesian aquifers in Texas: Econ. Geology, vol. 40, no. 3, pp. 193-226.

Jacob, C. E., 1940, On the flow of water in an elastic artesian aquifer: Am. Geophys. Union Trans., 1940, pt. 2, pp. 574-586.

Jacob, C. E., 1941, Coefficients of storage and transmissibility obtained from pumping tests in the Houston district, Tex.: Am. Geophys. Union Trans., 1941, pt. 3, pp. 744-756.

Jacob, C. E., 1946, Drawdown test to determine effective radius of an artesian well: Am. Soc. Civil Eng. Proc., vol. 72, no. 5, pp. 629-646.

Theis, C. V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., 1935, pt. 2, pp. 519-524.

Wenzel, L. K., 1942, Methods for determining permeability of water-bearing materials: U. S. Geol. Survey Water-Supply Paper 887.

Wenzel, L. K., and Greenlee, A. L., 1944, A method for determining transmissibility and storage coefficients by tests of multiple well systems: Am. Geophys. Union Trans., 1943, pt. 2, pp. 547-560.