SENSITIVITY IN RISK ASSESSMENT FOR THE YUCCA MOUNTAIN HIGH-LEVEL NUCLEAR WASTE REPOSITORY SITE: THE MODEL AND THE DATA

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INTRODUCTION

The final report for the research in the area of "Sensitivity in Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data" includes the following contributions:

A. **Articles**

(1) Ho, C.-H. 1993. Time Regimes in the Volcanic History of Vesuvius: 1631-1944, Bulletin of Volcanology (accepted).

(2) Ho, C.-H. 1993. Sensitivity in Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data (to be submitted in the near future).

B. **Paper presented**

"Comments on the Preliminary Draft of Los Alamos National Laboratory on the Status of Volcanic Hazard Studies for the Yucca Mountain Site Characterization Project," presented at the meeting of DOE-NRC Technical Exchange on Volcanism Studies held in Las Vegas on June 9, 1993.

FUTURE WORK: A Compound Power-Law Model for Volcanic Eruptions

Future work will concentrate on the following:

(1) A more general model for volcanism will be developed where the recurrence rates of a group of volcanoes are distributed according to nonhomogeneous Poisson processes having Power Law intensity functions with gamma distributed intensity parameter.

(2) Development of control charts for stochastic phenomena, which have general application worldwide, will continue.

(3) Several major papers will be prepared and submitted for publication during the grant period.

1 BACKGROUND

In the ongoing national debate on nuclear power as a source of electricity, a key issue is the disposition of the high level radioactive wastes produced in the process. At an earlier stage on this debate, Congress, aware of the importance of the waste issue, passed the Nuclear Waste Policy Act of 1982. This legislation required that the federal government develop a geologic repository for the permanent disposal of the high level radioactive wastes from civilian nuclear power plants. This waste consists primarily of spent nuclear fuel. Congress designated the Department of Energy (DOE) to implement the provisions of the act.

The Department of Energy established the Office of Civilian Radioactive Waste Management (OCRWM) in 1983 in response to the legislation and set about to identify potential sites. When OCRWM had selected three potential sites to study, Congress enacted the Nuclear Waste Policy Amendments Act of 1987, which directed the DOE to characterize only one of those sites, Yucca Mountain, in southern Nevada.

To characterize the site, the DOE must study in detail the natural environment and the various natural processes to which a proposed deep geologic repository might be subject. For a site to be acceptable, these studies must demonstrate that the site could comply with regulations and guidelines established by the federal agencies that will be responsible for licensing, regulating, and managing the waste facility. The regulations, which were promulgated to ensure the safety of the public, require that radiation will not be released above some established safe limit, determined by the Environmental Protection Agency (EPA), for at least 10,000 years after the repository is permanently sealed.

An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site (DOE, 1986) because the late Tertiary geologic history of southwestern Nevada has been dominated by volcanism and the consequent deposition of volcanic flows and tuffaceous rocks. Yucca Mountain, like most surrounding ranges, is composed dominantly of a series of Miocene ashflow tuff units and silicic volcanic rocks.

2 RELATED ISSUES

Yucca Mountain is located in the southcentral part of the Southwestern Nevada Volcanic Field (SNVF), a major volcanic province of the southern Great Basin first defined by Christiansen et al. (1977) and extended by Byers et al. (1989). Interested readers are referred to the papers of Byers et al. (1989) for the location of geographic features of the SNVF, and Crowe (1990) for the basaltic volcanic episodes of the Yucca Mountain region. Before developing formal results, it is useful to briefly review the controversy over some issues related to the volcanological studies at the Yucca Mountain region, straddling the southern corner of the Nevada Test Site (NTS). where nuclear materials have been handled for more than three decades.

2.1 Modeling Assumptions for the Recurrence Rate

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risks, and probabilities. There is a large and growing body of literature on probabilistic modeling for volcanism. Much of the debate in the literature is centered on the choice of distribution models (principally homogeneous Poisson vs. nonhomogeneous Poisson models).

Several probabilistic assessments of volcanic risk at Yucca Mountain are available (Crowe et al. 1982, Crowe and Perry 1989, Ho et al. 1991, and Ho 1991a, 1992). All rely on dividing the probabilistic risk assessment into two steps of estimating: (1) λ, the recurrence rate of volcanic activities near the Yucca Mountain region, and (2) p, the conditional probability of site disruption given a volcanic event. Crowe and his co-workers (1982, 1989) have estimated λ using a simple Poisson model. Ho (1990, 1991b) examines the applicability of the simple Poisson model for volcanic eruption forecasting. He notes that while the simple Poisson model can be used for modeling volcanic events from some volcanoes, it may not be appropriate in all cases. Therefore, Ho (1991ab, 1992) proposes a Weibull model which allows for waning or waxing trends in volcanism through time. A brief description of the technique is reviewed in Section 3.

2.2 The Eruptive History of the Basaltic Volcanism

An area of difficulty in reconstructing the eruptive history of the Quaternary basalt near the Yucca Mountain site is in determining whether each center formed in a single eruption (monogenetic) or multiple time-separate eruptions (polycyclic). Additionally, it is difficult to establish an age of eruptive activity for each center with a reasonable degree of accuracy. Small volume basalt centers have traditionally been assumed to be monogenetic centers (Wood 1980). However, detailed studies of the Lathrop Wells and Sleeping Butte centers (Wells et al. 1990; Crowe and Perry 1991) have raised the possibility that some basalt centers may form episodically (polycyclic volcanism).

The Lathrop Wells volcanic center is located 20 km south of the potential Yucca Mountain site, at the south end of the Yucca Mountain range. It has long been recognized as the youngest basalt center in the region. However, determination of the age and eruptive history of the center remain the subject of considerable debate. Isotopic ages between 3.8 and 0.3 Ma have been obtained from the cinder cone centers in Crater Flat by Turrin and Champion, (1991). However, Wells et al. (1990) have argued that the Lathrop Wells cone may be as young as 20 ka based on geomorphic and pedogenic characteristics as well as on the scatter of isotopic ages. Wells et al. (1990) further suggest that this center contains at least three discrete and temporally separate eruptive events that may have occurred over time spans of 1-10 ka, based on mapping of stratigraphic relations of tephra (volcanic debris) units here and elsewhere in the Basin and Range (Crowe et al., 1989). In contrast, ^{40}Ar/^{39}Ar age dating of two separate flow units in the Lathrop Wells volcanic center yields arithmetic means of ages of 183 ± 21 and 144 ± 35 ka (Turrin et al., 1991). On the basis of this dating and as yet unpublished K/Ar dates, Turrin et al. (1991) conclude that there were two eruptive events at Lathrop Wells, dated at 136 ± 8 ka and 141 ± 9 ka. They speculate that the time interval between flows may be less than 100 years because field mapping and paleomagnetic data indicate remanent magnetization directions only a few degrees apart for the two flow units. Differences in remanent magnetization directions can be accounted for by secular (temporal) variation of the earth's magnetic field, the rates of which have been calibrated in other volcanic fields at approximately 4° per 100 years. Thus, they interpret the nearly identical remanent directions in the two Lathrop Wells flow units to imply a short duration (< 100 years) of eruptive activity. However, this interpretation of the paleomagnetic data is controversial. Because both remanent directions are very similar to the time-averaged geomagnetic field in the study area, these directions could represent equally well eruptions separated by 100 years, 10 ka, 100 ka, or 1 Ma. Therefore, it is not surprising that data are inconsistent at this early stage of site characterization studies.

2.3 Structural Controls of Basaltic Volcanic Activity

Crowe and Perry (1989) describe the distribution of volcanic centers, emphasizing a southwest stepping of volcanism between 6.5 and 3.7 Ma. They describe a recurrence pattern of basaltic events where new eruptive sites are marked by probable coeval clusters of centers. These clusters appear to be of similar age within the limits of K-Ar age determinations. They note that all basalt centers of the youngest episode of volcanism, except the basalt of Buckboard Mesa, occur in a narrow northwest trending zone. They named this zone the Crater Flat volcanic zone (CFVZ, see Fig. 1). Crowe and Perry (1989), and Crowe (1990) suggest a southwest migration of basaltic volcanism in the Yucca Mountain area based on this structural parallelism, a pattern that may reflect an earlier southwest migration of silicic volcanism in the Great Basin. Smith et al. (1990) examine the spatial and temporal patterns of Post-6 Ma volcanism in the southern Great Basin. They describe the area of most recent volcanism (AMRV, see Fig. 1) near Yucca Mountain as an area enclosing all known post-6 Ma volcanic centers in the region and examine the implications of the information for an assessment of volcanic risk. Smith et al. (1990, 1993) provide a different point of view of the migration trends of volcanism in the Yucca Mountain region. They suggest that the structural control of basaltic volcanism should be evaluated at two scales. First, the control of large-scale regional structures (strike-slip faults, detachments) and volcano alignments related to these structures should be evaluated. Second, control of structures on and adjacent to Yucca Mountain and volcano alignments related to these structures should be evaluated. Models for structural control because of the different scales of geologic structures may be different. For example, northwest striking structures may result in a regional alignment of Pliocene and Quaternary cones in a northwest direction. But, at the scale of Yucca Mountain, northeast striking structures control the alignment of volcanoes (Smith et al., 1990). Although both models may be supported by the data, a judgment must be made as to which model is most appropriate for risk studies at Yucca Mountain. In contrast to the work of Crowe et al. (1982), Ho (1992) has recently incorporated numerically the possibility that the sites of future volcanism may be controlled by specific segments of structures developed by Smith et al. (1990) into the site disruption parameter, **p**. Point estimation and prior determination of **p** are reviewed in Section 4.

2.4 Counts of Volcanic Events

In order to estimate the recurrence rate of the volcanism and the volcanic risk to the repository, the definition of a single event has to be addressed. An accurate count of the number of eruptions is possible for volcanoes with a complete historical record. As no historical record is available for the Yucca Mountain region, identifying the number of eruptions depends on clear understanding of eruptive processes and reliable dating technique. Crowe et al. (1983) indicate that a main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone. Therefore, Ho (1991a, 1992) attributes a single date to the cluster and creates a separate event with that date for each main cone in the cluster, using this definition of a single eruption from Crowe et al. (1983). An alternative definition of a single event would be a single cluster of volcanic centers, because one may argue that all main cones in a cluster could arise from the same eruption. Also, a simple volcanic center could be a third possibility to define a single event if it is geologically meanful.

2.5 Rationale

It is useful to perform sensitivity study of volcanic risk assessment for the Yucca Mountain site because the controversy over the important issues we have briefly reviewed. The following development is to account for some significant geological factors raised by experts. Specifically, we will concentrate on the treatment of the model and the data.

3. MODELS FOR VOLCANIC ACTIVITY

3.1 Simple Poisson Process

The application of statistical methods to volcanic eruptions is put onto a sound analytical footing by Wickman (1966, 1976) in a series of papers that discuss the applicability of the methods and the evaluation of recurrence rates for a number of volcanoes. Wickman observes that, for some volcanoes, the recurrence rates are independent of time. Volcanoes of this type are called "Simple Poissonian Volcanoes." Theoretically, the probability model for simple Poissonian volcanoes is derived from the following assumptions:

Volcanic eruptions in successive time periods of length t for each period are independent and should follow a Poisson distribution with a constant mean (average rate) $\mu = \lambda t$, where λ is the recurrence rate in unit time and is assumed to be constant throughout the entire life of the volcanic activity.

If λ is assumed constant over t, the process is referred to as a homogeneous Poisson process (HPP). Since λ is constant and the increments are independent, it turns out that one does not need to be concerned about the location of the observation time interval, and an HPP is applicable for any interval of length t, $[s, s+t]$, $\mu = \lambda t$. That is, regardless of the interval chosen, the variable remains Poisson with the appropriate mean. If events occur according to a Poisson process with parameter λ, then the waiting time until the first occurrence, T_1, follows an exponential distribution, $T_1 \sim \text{Exp}(\theta)$ with $\theta = 1/\lambda$. Furthermore, the times between consecutive occurrences are independent exponential variables with the same mean time between occurrences, $1/\lambda$. The assumption of a constant recurrence rate λ suggests that the volcanism, which depends on the availability of magma and a functioning triggering mechanism, as well as on their mutual interaction, is relatively uniform and does not get "exhausted" by loss of gases or for other reasons.

Suppose we assume that the successive volcanic eruptions at the Yucca Mountain region follow a simple Poisson process. Let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$. The following theoretical results are useful for this study:

1. The maximum likelihood estimator for the recurrence rate λ is (Ho et al. 1991):

$$\hat{\lambda} = n/t.$$

2. The number of future eruptions, N, during $[t, t + t_0]$ would be distributed as a homogeneous Poisson random variable with constant rate λt_0,

$$P(N = k) = \exp\,[-\lambda t_0][\lambda t_0]^k/k!,\ k = 0, 1, \cdots$$

3.2 Weibull Process

If the volcanism is waning or waxing, the model should be generalized to allow λ to be, respectively, a decreasing or increasing function of t. More generally, one might want to allow the recurrence rate to be an arbitrary nonnegative function of t. Specifically, for volcanism, Ho (1991a,b) considers a nonhomogeneous Poisson process (NHPP) with intensity function $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ for $\beta, \theta > 0$. The parameters β and θ are sometimes referred to as shape and scale parameters, respectively. Because $\lambda(t)$ is the failure rate for the Weibull distribution, the corresponding process has been called the Weibull process (WP). Goodness-of-fit, maximum likelihood (ML) estimates of β and θ, confidence intervals, and inference procedures for this process are presented in Bain and Engelhardt (1980), Bassin (1969), Crow (1974, 1982), Finkelstein (1976), and Lee and Lee (1978). A WP is appropriate for three types of volcanoes: increasing-recurrence-rate ($\beta > 1$), decreasing-recurrence-rate ($\beta < 1$), and constant-recurrence-rate ($\beta = 1$). This generalized model can be considered a goodness-of-fit test for an exponential model ($\beta = 1$) of the volcanic inter-event time, which is equivalent to a homogeneous Poisson model of the events. In a simulation study, Bain et al. (1985) conclude that the test which is derived as an optimal test for the WP also is rather powerful as a test of trend for general

NHPP's. In other words, the test is "robust" against other model assumptions. This is the rationale of our choice of a WP to amend a simple Poisson model which neglects the time trend of the volcanic activities. Again, suppose we assume that the successive volcanic eruptions at the Yucca Mountain region follow a WP. For a time-truncated WP, let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at time $0 < t_1 < t_2 < \cdots < t_n$. Some useful theoretical results to be used later are summarized as follows:

1. The maximum likelihood estimates (MLE) of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n / \sum_{i=1}^{n} \ln(t/t_i)$$

$$\hat{\theta} = t / n^{1/\hat{\beta}}$$

2. If a WP is assumed during the observation time period $[0, t]$, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ at time t. In the application of the WP to volcanic eruptive forecasting, the estimate of $\lambda(t)$ is of considerable practical interest since $\lambda(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. Crow (1982) derives the MLE for $\lambda(t)$ as

$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\theta})(t/\hat{\theta})^{\hat{\beta}-1} = n\hat{\beta}/t.$$

3. Using the same WP, the number of occurrences, N, in time $[t, t + t_0]$, is a Poisson random variable,

$$P(N = k) = \exp[-m(t_0)][m(t_0)]^k/k!; \quad k = 0, 1, \cdots$$

where

$$\begin{aligned} m(t_0) &= \int_t^{t+t_0} \lambda(s)ds \\ &= [(t+t_0)^\beta - t^\beta]/\theta^\beta \end{aligned}$$

(m obviously depends on t but our notation suppresses t because t is the known observation period.)

4. MODELING OF VOLCANIC DISRUPTION

4.1 Classical Approach

If we consider the fact that not every eruption would result in disruption of the repository, and let **p** be the probability that any single eruption is disruptive, then the number of occurrences of such a disruptive event $X(t_0)$ in $[0, t_0]$ also follows a homogeneous Poisson random variable with constant rate (Meyer, 1965, p. 156). An important element in assessing the suitability of the site is an assessment of the potential for future volcanic disruption of the repository. Therefore, the probability of at least one disruptive event during the next t_0 years is of considerable practical interest and is quoted as "risk." In a classical statistical analysis, we would use the Poisson probability distribution formula,

$$\begin{aligned} \text{risk} &= \Pr(\text{at least one disruptive event before time } t_0) \\ &= 1 - \exp\{-\lambda \mathbf{p} t_0\} \end{aligned}$$

for an HPP. And

$$\text{risk} = 1 - \exp\{-m(t_0)\mathbf{p}\}$$

for a WP. Point or interval estimates for the risk can be obtained based on those of $\mathbf{p}$, λ, and $m(t_0)$.

4.2 Bayesian Approach

For the Bayesian approach, we consider λ and $m(t_0)$ to be fixed for both the HPP and the WP, and we permit prior distribution for $\mathbf{p}$. The prior distribution, $\pi(p)$, of $\mathbf{p}$ expresses our beliefs regarding the numerical values of $\mathbf{p}$. This would incorporate uncertainty about the probability of repository disruption $\mathbf{p}$ that are eventually averaged out as shown in the following equations. In this case, using the model of constant λ

$$\text{risk} = 1 - \int_p \exp\{-\lambda p t_0\}\pi(p)dp$$

for the HPP. And,

$$\text{risk} = 1 - \int_p \exp\{-m(t_0)p\}\pi(p)dp$$

for the WP. The technical machinery (Bayesian approach) involved in the above equations would support much more informative answers if the prior distribution $\pi(p)$ is adequately chosen.

4.3 Point estimation and prior determination of p

Crowe et al. (1982) assume that every eruption has the same probability of repository disruption $\mathbf{p}$, and provide a point estimate for $\mathbf{p}(= a/A)$. The calculations are based on a fixed value of a (= area of the repository estimated at 6-8 km^2), and several choices of A. (An area, range from 1,953 km^2 to 69,466 km^2, corresponds closely to a defined volcanic province and satisfies the requirement of a uniform value of λ.) Results of point estimates of **p** taken from publications ranging from 1980 to 1992 are listed in Table 7.1 of Crowe et al. (1993). The values range from 8×10^{-2} to 1.1×10^{-3}. We shall use these two bounds of **p** for the classical approach in the sensitivity data analysis.

We now turn to the description of the prior density for the Bayesian approach (Ho, 1992). Since the permissible range of **p** is $0 < p < 1$, without use of expert opinions regarding the geological factors at NTS, a natural choice for $\pi(p)$ is a noninformative prior. For instance, $U(0,1)$ (uniform 0, 1) assumes an average of 50% "direct hit," which is unrealistically conservative (overestimation). Ho (1992) settles on one particular prior based on the geological structure of the volcanic centers at NTS.

According to Smith et al. (1990), the area of most recent volcanism (AMRV) includes all known post-6-Ma volcanic complexes in the Yucca Mountain area and encompasses the four volcanic centers in Crater Flat, the Lathrop Wells cone, several centers in southeast Crater Flat, two centers at Sleeping Butte, and a center at Buckboard Mesa within the moat of the Timber Mountain Caldera. They conclude that future volcanic events in the Yucca Mountain area will be associated with Quaternary centers in Crater Flat, at Sleeping Butte, or at the Lathrop Wells cone (see Fig. 2). Based on their assumption, a future eruption may occur either to the north-northeast or south-southwest of an existing cone or group of cones. They show high risk zones within the AMRV in Fig. 3 by placing two rectangles on each group of Quaternary cones. The proposed high-level nuclear waste repository at Yucca Mountain falls within the larger high-risk Lathrop Wells rectangle and just to the east of the high-risk zones constructed for the Crater Flat chain as described in Fig. 3. The dimensions of the larger Lathrop Wells rectangle are 50 km long and 3 km wide as determined by analog studies of Pliocene volcanic centers in the Fortification Hill field (Lake Mead area, Arizona and Nevada) and the Reveille Range (south-central Nevada). The lower half of this rectangle is outside the AMRV.

Now, using the idea of Crowe et al. (1982), assume there is no heterogeneity with respect to disruptiveness in the upper-half of the rectangle that encloses the repository (the eruptions to the south-southwest of the Lathrop Wells cone are outside the AMRV, and have near zero probability of disrupting the site). So, given $A = 75$ km^2 (=half of the area of the rectangle), $a = 8$ km^2 (area of the repository), we obtain $p = a/A = 8/75$. Therefore, a more informative prior, $U(0, 8/75)$, which assumes 8/75 as the upper limit for p seems to be more suitable. We shall conduct all Bayesian analysis based on this prior which is developed from the geological structure of the volcanic centers at NTS.

5. DATA

The following is a summary of field and chronology data using the most current information from site characterization studies (Crowe et al. 1993).

Pliocene volcanic events in the Yucca Mountain region include:

1. 4.6 Ma Centers: **Basalt of the Thirsty Mesa**. This is a lava mesa formed from three coalesced vents. It is treated as one or three events with an age of 4.6 Ma (Champion 1992).
2. 4.4 Ma Center: **Basalt of the Amargosa Valley**. This volcanic event is represented by the aeromagnetic anomaly located a few kilometers south of the town of Amargosa Valley.
3. 3.7 Ma Centers: **Basalt of southeast Crater Flat.** This Pliocene unit consists of five centers representing one to five events. The age of the centers is assumed to be well dated at 3.7 Ma.
4. 2.9 Ma Centers: **Basalt of Buckboard Mesa**. This consists of one center or event forming a lava mesa and small cone in the moat zone of the Timber Mountain caldera.

Quaternary events in the Yucca Mountain region include:

1. 1.1 Ma Centers: **Quaternary basalt of Crater Flat**. These are treated by Crowe et al. (1993) as four individual centers. Smith et al. (1993) suggest that at least six centers must now be considered in the calculation from the Quaternary basalt of Crater Flat: NE Little Cone, SW Little Cone, Black Cone, Northern Cone, Red Cone 1, Red Cone 2.

2. 0.38 Ma Centers: **Basalt of Sleeping Butte**. These are treated as two individual centers clustered on a northeast-trend 45 km northwest of the Yucca Mountain site.

3. 0.1 Ma Centers: **Lathrop Wells Center**. This is treated as a single event center formed in two pulses of activity, one at about 100 to 140 ka,the other at > 40 ka. The existence of a potential young volcanic event (10 ka) at the center remains controversial but possible (Crowe et al. 1993).

Another key issue in the sensitivity analysis is to specify the observation period, $[0, t]$, in modeling the volcanic history at NTS. Most of the volcanic risk assessment studies in the Yucca Mountain area are centered around the post-6-Ma (Pliocene and younger) and Quaternary (< 1.6 Ma) volcanism (Crowe et al., 1988, 1989, Smith et al., 1990; Wells et al., 1990). We shall use the above dates to estimate the recurrence rate of volcanism during the following two observation periods: Pliocene and younger (< 6.0 Ma), and Quaternary (< 1.6 Ma). Therefore, let the beginning of the Pliocene period ($\doteq 6.0$ Ma) be time zero, so $t = 6.0$ Ma. For the study on Quaternary volcanism, $t = 1.6$ Ma. Prediction of future volcanic activities (volcanic eruption and site disruption) will focus on the entire life of the repository (10^4 years is recommended as the required isolation period during which radioactive waste may decay to an accetable level). Thus, we shall evaluate the risk with $t_0 = 10,000$.

6. DATA ANALYSIS

Three types of models are considered in the following sensitivity analysis. The first model (HPP) assumes that both past and future volcanic activities follow an HPP. The second model (WP-HPP) uses a WP to estimate the instantaneous recurrence rate based on the historical data at NTS. The model then switches from a WP of past events to a predictive HPP (a constant rate for future events). The third model (WP) assumes that the prior historical trend based on a WP would continue for future activities. Risks(at least one disruptive event before time t_0) using both classical and Bayesian approaches are evaluated based on the data for the following two observation periods: Pliocene and younger (t = 6.0 Ma), and Quaternary (t = 1.6 Ma).

As we have mentioned, another key issue in the site characterization studies is the disagreement over age-dating of the rocks and counts of volcanic events. The following treatment of the data is to account for some significant differences raised by experts. The dates (in Ma) summarized from Section 5 are: 4.6 (1 to 3 events), 4.4, 3.7 (1 to 5 events), 2.9, 1.1 (4 to 6 events), 0.38 (2 events), 0.1, 0.01 (this remains controversial but possible). Combinations of various counts at volcanic centers of controversy and inclusion (or exclusion) of the youngest date (= 0.01) generate 90 ($= 3 \times 5 \times 3 \times 2$) different data sets (Pliocene and younger volcanism). The sensitivity analysis are performed for each data set and only the minimum and the maximum risks for each model are summarized in Table 1 (Quaternary volcanism) and Table 2 (Pliocene volcanism).

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Table 1. Results of the sensitivity analysis for the proposed Yucca Mountain Repository site based on the data of Quaternary volcanism

Model	Recurrence Rate (min, max)	Risk		
		Classical $p = 1.1 \times 10^{-3}$	Classical $p = 8 \times 10^{-2}$	Bayesian
HPP	$(4.38 \times 10^{-6}, 6.25 \times 10^{-6})$	$(4.81 \times 10^{-5}, 6.87 \times 10^{-5})$	$(3.49 \times 10^{-3}, 4.99 \times 10^{-3})$	$(2.33 \times 10^{-3}, 3.33 \times 10^{-3})$
WP-HPP	$(5.83 \times 10^{-6}, 8.23 \times 10^{-6})$	$(6.40 \times 10^{-5}, 9.06 \times 10^{-5})$	$(4.65 \times 10^{-3}, 6.56 \times 10^{-3})$	$(3.10 \times 10^{-3}, 4.38 \times 10^{-3})$
WP	$(5.83 \times 10^{-6}, 8.23 \times 10^{-6})$	$(6.41 \times 10^{-5}, 9.06 \times 10^{-5})$	$(4.65 \times 10^{-3}, 6.57 \times 10^{-3})$	$(3.10 \times 10^{-3}, 4.38 \times 10^{-3})$

Table 2. Results of the sensitivity analysis for the proposed Yucca Mountain Repository site based on the data of Pliocene and younger volcanism

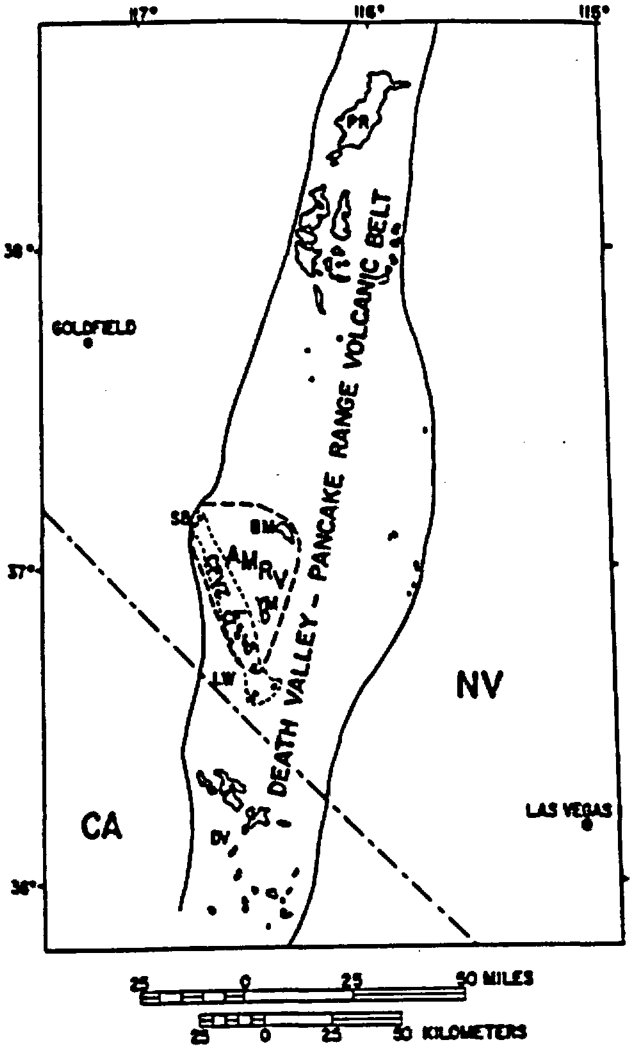
Model	Recurrence Rate (min, max)	Risk		
		Classical $p = 1.1 \times 10^{-3}$	Classical $p = 8 \times 10^{-2}$	Bayesian
HPP	$(1.83 \times 10^{-6}, 3.33 \times 10^{-6})$	$(2.02 \times 10^{-5}, 3.67 \times 10^{-5})$	$(1.47 \times 10^{-3}, 2.66 \times 10^{-3})$	$(9.77 \times 10^{-4}, 1.78 \times 10^{-3})$
WP-HPP	$(3.41 \times 10^{-6}, 5.67 \times 10^{-6})$	$(3.75 \times 10^{-5}, 6.24 \times 10^{-5})$	$(2.72 \times 10^{-3}, 4.53 \times 10^{-3})$	$(1.82 \times 10^{-3}, 3.02 \times 10^{-3})$
WP	$(3.41 \times 10^{-6}, 5.67 \times 10^{-6})$	$(3.75 \times 10^{-5}, 6.24 \times 10^{-5})$	$(2.72 \times 10^{-3}, 4.53 \times 10^{-3})$	$(1.82 \times 10^{-3}, 3.02 \times 10^{-3})$

Figure Captions

Fig. 1. The proposed area of most recent volcanism (AMRV) is outlined by a heavy dashed line and includes the Lathrop Wells cone (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM) and volcanic centers within Crater Flat (CF). For comparison the Crater Flat Volcanic Zone (dashed line) and the Death Valley-Pancake Range Volcanic Belt (solid line) are shown. PR = Pancake Range. YM = proposed drift perimeter at Yucca Mountain. DV = Death Valley. (Source: Smith et al. 1990, figure 2)

Fig. 2. Generalized geologic map of Crater Flat volcanic field area and boundary of proposed radioactive waste repository; inset map shows location of the Crater Flat volcanic field. (Source: Wells et la. 1990, figure 1)

Fig. 3. Map outlining the AMRV (dashed line) and high-risk zones (rectangles) in the Yucca Mountain (YM) area that include Lathrop Wells (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), volcanic centers within Crater Flat (CF). (Source: Smith et al. 1990, figure 7)



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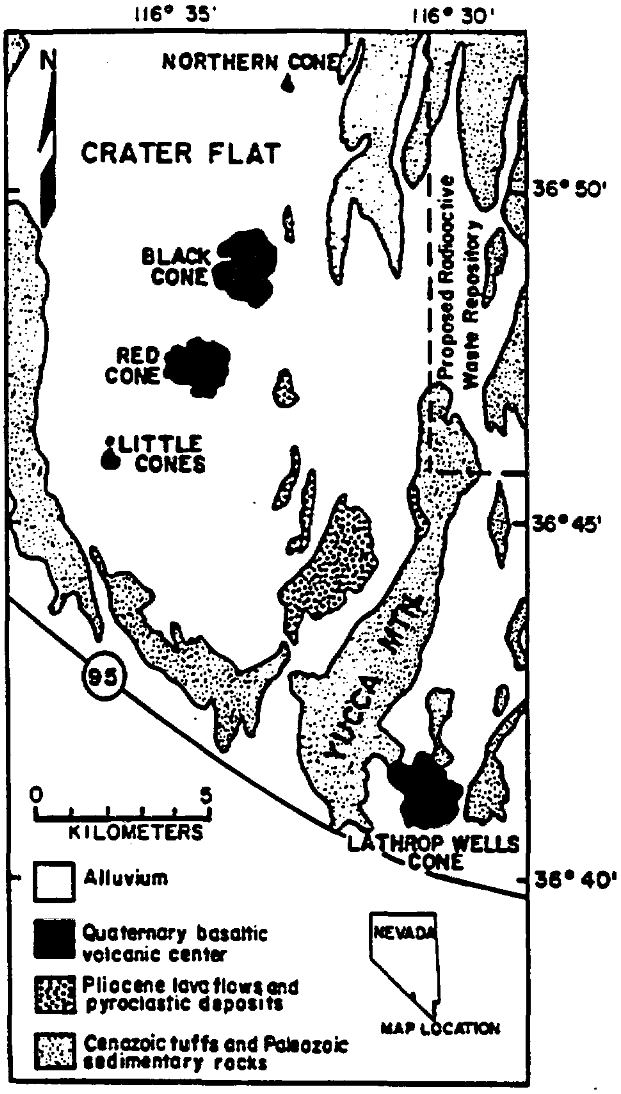

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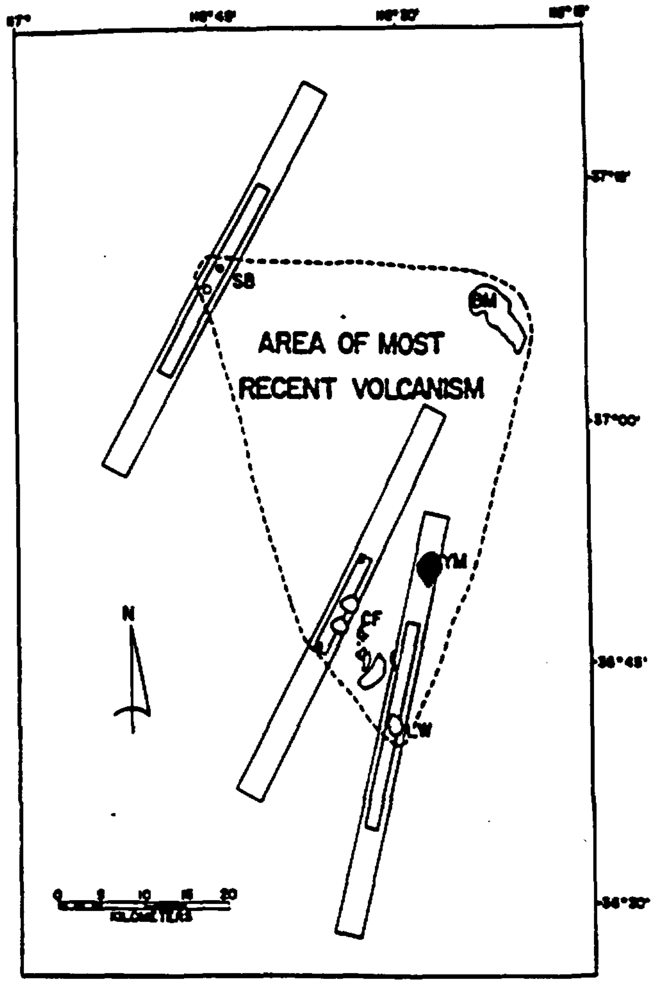

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Time Regimes in the Volcanic History of Vesuvius: 1631-1944 (*)

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ABSTRACT

Wickman (1966) uses a series of repose states characterized by time-independent rate parameters (recurrence rates) to describe the repose-period patterns of Vesuvius during the period of 1631-1944. Wickman's assumptions are verified by Carta et al. (1981), who divide the periods into a series of eruptive cycles. Their methods are based on subjective grounds, mostly on the basis of cumulative plots. In this article, we briefly review a different approach developed by Ho (1992) for time regime identification based on the original chronological order of the eruptions. A time regime of a volcano is defined as a time period in which the volcano behaves as a simple Poissonian volcano. We then apply the procedure to the eruptive history of Mount Vesuvius based on the record (1631-1944) constructed by Carta et al. (1981). The application shows schematically that the volcanic time series of Vesuvius are modeled as a sequence of time regimes with the duration of each time regime being a random variable distributed according to an exponential distribution, which is equivalent to a simple Poisson model of the events (volcanic eruptions) within each time regime.

INTRODUCTION

Vesuvius is a volcano on the shore of the Bay of Naples in central Italy. Located in an area of Europe that has been populated for almost 3000 years, more is known of the eruptive history of Vesuvius than of almost any other volcano (Bullard 1984). Since AD 79, Vesuvius has been rather active, with major eruptions occurring in 472, 512, 685, 993, 1036, 1049, 1138, 1139, and 1631. The eruption of 1631 is particularly significant. Vesuvius has been in a constant state of activity since 1631, with noteworthy large eruptions in 1779, 1794, 1822, 1838, 1850, 1872, 1906, and 1944. There has been no major activity since 1944, although that eruption did not seem to signal the end of a cycle.

Although every volcano has an individual repose-period pattern, there are, nevertheless, several general types of patterns (Wickman, 1966, 1976), which make long-term forecasting possible for volcanoes with simple extreme patterns. Wickman observed that, for some volcanoes, the recurrence rates were independent of time. These volcanoes were called "simple Poissonian volcanoes". Wickman also uses a series of repose states characterized by time-independent rate parameters to describe the repose-period patterns of Vesuvius during the period of 1631-1944. In other words, Vesuvius is modeled as a sequence of activity states (Markov chains) with the duration of each state being a random variable distributed according to an exponential distribution. Wickman's assumptions are verified by Carta et al. (1981), who divide the periods into a series of eruptive cycles. These cycles are characterized by four states: repose (R), persistent activity (A), intermediate eruption (IE), and final eruption (FE). To model the duration of the states, Carta et al. (1981) favor Wickman's hypothesis that they are random variables from exponential distributions of form $f_s(x) = \lambda_s e^{-\lambda_s x}$ where s labels the states. To determine λ_s, Carta et al. (1981) suggest two methods: $\bar{x} = s^2 = 1/\lambda_s$, where $\bar{x}$ is the sample mean of repose periods, and s^2 is the sample variance; and minimizing $F(t) - \int_0^t f(x)\mathrm{d}x$, where $f(x)$ is the exponential probability density function, and $F(t)$ is the cumulative distribution function of the data. However, the states identified by Wickman (1966) and Carta et al. (1981) are based on subjective grounds, mostly on the basis of cumulative plots. Although in some cases the cumulative plots can be very helpful in the visual detection of trends, they do not lead to quantitative and objective results, and therefore do not guarantee a fully scientific approach to the problem.

In this article, we shall use the procedure developed by Ho (1992) to partition the volcanic eruptive history of Vesuvius for the period 1651-1944 into several time regimes. Each time regime represents a time period in which Vesuvius behaved as a simple Poissonian volcano. A brief description of the technique is reviewed in the next section.

METHOD

We wish to distinguish between the variation inherent in the repose times observed and the extraordinary variation that signals a real change in the eruptive time-history of Vesuvius. The essence of Ho's method is statistical process control, which is a sophisticated concept because it recognizes that variability will be present and requires only that the pattern of variability remain the same. A variable is said to be in control when it continues to be described by the same distribution when observed over time. Control charts are the mechanism for determining if a process is in control. Basically, a statistic (a number calculated from the observations in a sample) is calculated for each point in time-sequential data. The value of this statistic is plotted over time. If the points all lie between two control limits, the process is considered to be in control. An out-of-control "signal" occurs when a point falls outside the limits. This is attributed to a new time regime. Therefore, the point immediately preceding that one marks the boundary between two time regimes and is called a "change-point." The control limits are determined from the distribution of the test statistic. Hence, the control chart is closely related to hypothesis testing. The null hypothesis (H_0) is that the process is in control. A type I error occurs if an in-control process produces a test point outside the control limits. Analogous to hypothesis testing, the control limits can be chosen to make the probability of type I error (α) reasonably small.

Ho (1992) uses a Weibull process for his procedure. A Weibull distribution WEI(θ, β) may be regarded as a generalization of the exponential distribution EXP(θ), where $\beta > 1$ means increasing-occurrence-rate, $\beta < 1$ decreasing-occurrence-rate, and $\beta = 1$ constant-occurrence-rate. Note that $\beta = 1$ is the exponential distribution. Ho concludes that the generalized model can be considered a goodness-of-fit test for an exponential model ($\beta = 1$) of the volcanic inter-event times, which is equivalent to a homogeneous Poisson model of the events.

The CSLR Procedure

Let $t_1, \cdots, t_n$ be the first n successive times of eruptions of a volcano. These times are measured from the beginning of the observation period (cumulative length of time over which the eruptions occur), so $t_1 < t_2 < \cdots < t_n$. The first step in examining eruptive process is to plot the statistic $[= 2\sum_{i=1}^{n-1} ln(t_n/t_i)]$ against the time order in which the measurements were recorded. If the eruption process is stable over time, the observed test statistic, $2\sum_{i=1}^{n-1} ln(t_n/t_i)$, should continue to be described by a chi-square distribution with $2n-2$ degrees of freedom, and the $(1-\alpha)100\%$ control limits are

$$LCL_\alpha = \text{lower control limit} = \chi^2_{\alpha/2}(2n-2)$$

$$UCL_\alpha = \text{upper control limit} = \chi^2_{1-\alpha/2}(2n-2),$$

where $\chi^2_{\alpha/2}(2n-2)$ is the $100\alpha/2$ percentile of a chi-square distribution with $2n-2$ degrees of freedom. The control limits are readily available from a table of the chi-square distribution. The same control limits are used for every stage of time regime identification for any volcano of interest. Therefore, this technique can be applied to other volcanoes in exactly the same setting as for the Vesuvius data demonstrated in this article. Since it requires at least two repose times for the statistical process control at each stage, Ho (1992) defines the cumulative sums of log ratio (CSLR) by

$$S_2 = 2\ ln(t_2/t_1)$$

$$S_3 = 2\ [ln(t_3/t_1) + ln(t_3/t_2)] = 2\sum_{i=1}^{2} ln(t_3/t_i)$$

$$\vdots$$

$$\begin{aligned} S_\ell &= 2\ [ln(t_\ell/t_1) + \cdots + ln(t_\ell/t_{\ell-1})] = 2\sum_{i=1}^{\ell-1} ln(t_\ell/t_i) \\ &= S_{\ell-1} + 2(\ell-1)ln(t_\ell/t_{\ell-1}) \end{aligned}$$

These cumulative sums are plotted over time. That is, at time ℓ of the ith stage, a point is plotted at height S_ℓ. At the current time point r in the current stage i, the plotted points are $(2, S_2)_i$, $(3, S_3)_i, \cdots, (r, S_r)_i$.

If at current time r, either $S_r \leq \chi^2_{\alpha/2}(2r-2)$ or $S_r \geq \chi^2_{1-\alpha/2}(2r-2)$, the process is judged to be out of control based on $(1-\alpha)100\%$ control limits. The first inequality suggests the process has shifted to an increasing time trend and thus a different regime has started at time $r-1$. Similarly, the second inequality suggests the process has shifted to a decreasing time trend. In either case, the $(r-1)$th eruption is identified as a change-point, which is the boundary point of two different time regimes. Therefore, the $(r-1)$th time point is regarded as time zero for the search of the next change-point. The control charting procedure continues until no more significant points can be found or until the size of the data set becomes too small (minimum = 2). Each of the identified regimes then belongs to a simple Poisson process without further goodness-of-fit testing. In other words, the volcano behaves as a simple Poissonian volcano in each time regime. Thus, using this technique, a volcano which has only one time regime in the entire history of eruptions is a simple Poissonian volcano.

DATA

Carta et al. (1981) reconstruct the history of Vesuvius between 1631 and 1944. Data are arranged in the form of a chronological succession of states with duration and uncertainty of each state. More precisely, the states considered in Carta et al. (1981) are:

a) Repose (R): the volcanic conduit is obstructed (at least at ground level), the only apparent activity being fumarolic emissions;

b) Persistent activity (A): the conduit is open and a conelet is built within the central crater by a continuous outflow of lava; lava emission is generally restricted to the crater area;

c) Intermediate eruption (IE): the activity is pertinent to a single physical state, with conspicuous effusions of lava outside the crater area and/or strombolian activity; possible opening of subterminal vents; this eruptive state is not followed by a repose;

d) Final eruption (FE): huge volumes of lava and pyroclastics are emitted in very short time; in many cases a mushroom plume is formed; final eruptions are extremely violent and may cause serious damage; they last only a matter of days and are always followed by the obstruction of the conduit and by a state of repose.

As noted by Carta et al. (1981), the intermediate eruptions and the final eruptions might be examples of a single physical state and differ in intensity and effects. Therefore, the two data sets to be used in this article for time regime identification are based on: (1) final eruptions only (FE), and (2) intermediate and final eruptions combined (IE & FE).

One more simplifying assumption must be made in treating eruptions as stochastic events in time. Although the onset date of an eruption is generally well-defined by the time when lava first breaks the surface, the duration is harder to determine because of such problems as slowly cooling flows or lava lakes and the gradual decline of activity. We adopt the same definition for repose time as defined by Klein (1982). We therefore, ignore eruption duration; instead, we take the onset date (in days) as most physically meaningful, and measure repose times from one onset date to the next. Thus, our definition of "repose time" differs from the classic one (a noneruptive period). Therefore, the data in Tables 1 and 2 represent the dates over which the eruptions begin based on the record of Vesuvius reconstructed by Carta et al. (1981).

Tables 1 & 2 here

RESULTS

All analyses are based on the 90% control limits.

Case 1: IE & FE

Five time regimes are identified. A visual interpretation for the time-trend information is best described by Figures 1 and 2. Fig. 1 (Lines 1-5) illustrates these time regimes. For comparison purposes, Line 6 in Fig. 1 shows the dot diagram of the eruptions in their original chronological order (in days). Regarding the first eruption (December 16, 1631) as time point 0, Fig. 2a shows that the lower 90% control limit is crossed at $(9, S_9)_1$ $[S_9 = 7.79 < 7.96 = \chi^2_{.05}(16)]$. Note that the rth point of the ith stage on the control chart is referenced as $(r, S_r)_i$. This suggests a shift to an increasing time trend, which is supported by Fig. 2a and by comparing Lines 1 and 2 of Fig. 1. As a result, the first change-point is identified as July 28, 1707, the time point immediately preceding April 26, 1712 (see Table 1). A new stage (time regime) starts at the first change-point with this point becoming time point 0 for the second stage. The next change-point is October 29, 1712 caused by a significant increasing time trend (see Fig. 2b and the insert in Fig. 1). The third change-point occurs at the eruption of May 20, 1737. The relatively long time interval between May 20, 1737 and the next eruption, October 25, 1751, signals a decreasing time trend, as the UCL is crossed at $(12, S_{12})_3$ as seen in Fig. 2c. The last change-point, March 28, 1766 is caused by a shift to an increasing time trend (Fig. 2d). All further events are in control (Fig. 2e). The change-points and eruption dates are summarized in Table 1.

Figure 1-2 here

Case 2: FE

Following the same argument as described in case 1 (IE & FE), two time regimes are identified (Fig. 3, Lines 1 and 2). The only change-point is identified at $(3, S_3)_1$ (May 25, 1698) as $(4, S_4)_1$ is out of control (see Figure 4). The change-point and eruption dates are given in Table 2.

Each time regime is considered to reflect an exponential distribution of inter-event times, hence a homogeneous Poisson distribution of events within that time regime. The estimated parameters, θ of the exponential distribution and $\lambda(= 1/\theta)$ of the corresponding Poisson process for each time regime, are shown in Table 3, as well as the starting and ending points of the identified time regimes (start of ith regime is end of $(i-1)$th regime). These values do not appear to follow any pattern.

Figure 3-4 & Table 3 here

DISCUSSION AND CONCLUSION

For Vesuvius data, we use the idea of statistical process control to distinguish between the variation inherent in the observed repose times and the extraordinary variation that signals a real change in the time regimes. The control chart procedure shows a point outside the control limits almost as soon as the process enters a new time regime. This procedure is an eruption by eruption procedure, which follows the original chronological order of the eruptions. The basis of the statistical process control mechanism is a simple Poisson process. The test statistics (CSLR) are sensitive to the locations, numbers, and relative sizes (to t_ℓ) of the ordered t_i's. If early sparse t_i's were accompanied later by dense t_i's toward t_n, then S_ℓ would be small, showing an increasing rate of recurrence through time, and vice versa. One may be surprised to see that a time regime has only two repose times (e.g., time regime #2 for the IE & FE data). Reading the graphs in Fig. 2b and the insert in Fig. 1, we are convinced that a long repose of 1,734 days after the eruption of July 28, 1707, contributes significantly to the breakdown of these time regimes based on the 90% control limits. However, if one takes the view that it doesn't make physical sense to have a simple regime including only two (or three, or more according to some geological criteria) repose times, the following alternatives may be appropriate.

(a) Combine each unacceptable time regime and an adjacent time regime having a similar recurrence rate into one. For example, based on the value of λ for time regimes 2 and 3 in Table 3 ($\lambda_2 = 0.00104$, $\lambda_3 = 0.00123$), one could have combined regimes 2 and 3 into a single time regime.

(b) Some other robust procedures need to be developed to prevent early detection of an out-of-control signal. For instance, one could be very conservative early by using large control limits and decrease the control limits during the course of the control charting procedure. Our efforts for future studies will be devoted to this goal.

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Figure Captions

Fig. 1. Dot Diagrams of five Vesuvius time regimes based on 90% control limits and the original time series data (IE & FE). The scale is enlarged in the insert for the time interval (in days) near the second change-point.

Fig. 2. Control charts for Vesuvius based on 90% control limits (IE & FE): (a) stage 1, (b) stage 2, (c) stage 3, (d) stage 4, (e) stage 5. The rth time point and the cumulative sums of log ratio (CSLR) at each stage as defined in the text are referenced as r and S_r.

Fig. 3. Dot Diagrams of two Vesuvius time regimes based on 90% control limits and the original time series data (FE).

Fig. 4. Control charts for Vesuvius based on 90% control limits (FE): (a) stage 1, (b) stage 2.

Table 1. Eruptions (IE & FE) of Vesuvius (1631-1944)

Time point (r) for stage					
1	2	3	4	5	Date
0					1631-12-16
1					1649-11-28
2					1660-07-03
3					1680-03-26
4					1694-04-13
5					1697-09-18
6					1698-05-25
7					1701-07-01
8	0				1707-07-28*
9	1				1712-04-26
	2	0			1712-10-29*
	3	1			1713-05-09
		2			1714-06-21
		3			1717-06-06
		4			1721-05-01
		5			1723-06-25
		6			1724-09-12
		7			1726-04-16
		8			1727-05-26
		9			1730-03-17
		10			1733-07-10
		11	0		1737-05-20*
		12	1		1751-10-25
			2		1754-12-02
			3		1759-03-31
			4		1760-12-23
			5	0	1766-03-28*
			6	1	1767-10-19
				2	1771-05-01
				3	1776-01-03
				4	1779-07-29
				5	1785-11-01
				6	1790-09-15
				7	1794-06-15
				8	1804-08-12

Table 1 continued

9	1804-11-22
10	1805-08-13
11	1805-10-14
12	1806-05-31
13	1812-01-01
14	1817-12-22
15	1822-02-22
16	1822-10-21
17	1831-09-21
18	1831-12-20
19	1832-02-21
20	1834-08-23
21	1839-01-01
22	1848-05-31
23	1850-02-06
24	1855-05-01
25	1858-05-28
26	1861-12-08
27	1868-11-15
28	1871-01-13
29	1872-04-26
30	1881-12-16
31	1885-01-19
32	1891-06-07
33	1895-07-06
34	1903-08-27
35	1906-02-03
36	1906-04-04
37	1929-06-03
38	1933-06-03
39	1936-04-28
40	1941-10-22
41	1944-03-17

* Change-point based on 90% control limits

Table 2. Eruptions (FE) of Vesuvius (1631-1944)

Time point (r) for stage		Date
1	2	
0		1631-12-16
1		1680-03-26
2		1694-04-13
3	0	1698-05-25*
4	1	1707-07-28
	2	1723-06-25
	3	1730-03-17
	4	1737-05-20
	5	1759-03-31
	6	1760-12-23
	7	1767-10-19
	8	1779-07-29
	9	1794-06-15
	10	1805-10-14
	11	1822-10-21
	12	1834-08-23
	13	1839-01-01
	14	1850-02-06
	15	1855-05-01
	16	1861-12-08
	17	1868-11-15
	18	1872-04-26
	19	1906-04-04
	20	1944-03-17

* Change-point based on 90% control limits

Table 3. Summary of regimes and regime parameters

Case 1: IE & FE

Regime	Dates(Starting)	θ	λ
1	12-16-1631	3452.1	.00029
2	7-28-1707	960.0	.00104
3	10-29-1712	815.4	.00123
4	5-20-1737	2107.8	.00047
5	3-28-1766 (current regime)	1585.4	.00063

Case 2: FE

Regime	Dates(Starting)		
1	12-16-1631	8089.0	.00012
2	5-25-1698 (current regime)	4489.0	.00023

θ = mean for the exponential distribution (in days)

λ = occurrence rate for the Poisson distribution

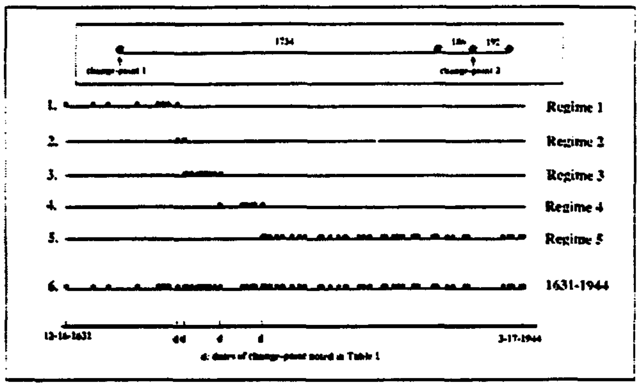

Figure 1. Dot Diagrams of five Vesuvious time regimes based on 90% control limits and the original time series data (IE & FE). The scale is enlarged in the insert for the time interval (in days) near the second change-point.

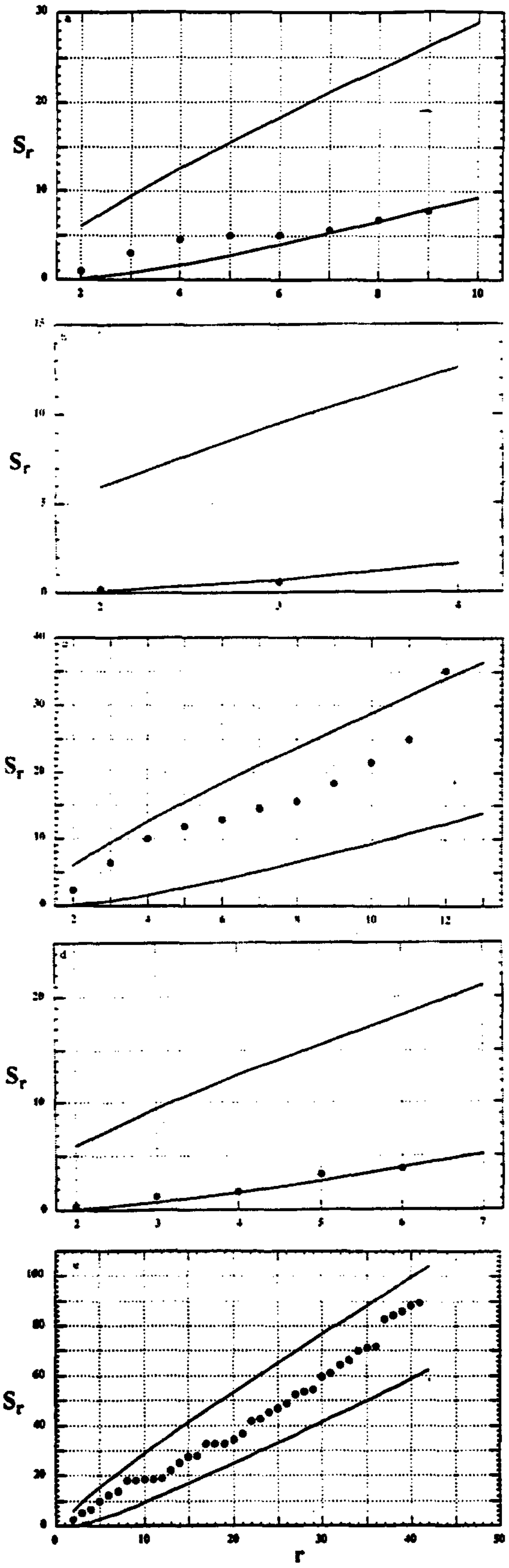

Figure 2. Control charts for Vesuvius based on 90% control limits (IE & FE): (a) stage 1, (b) stage 2, (c) stage 3, (d) stage 4, (e) stage 5. The *r*th time point and the cumulative sums of log ratio (CSLR) at each stage as defined in the text are referenced as *r* and S_r.

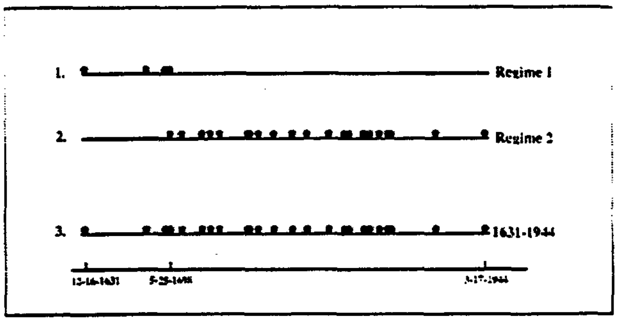

Figure 3. Dot Diagrams of two Vesuvius time regimes based on 90% control limits and the original time series data (FE).

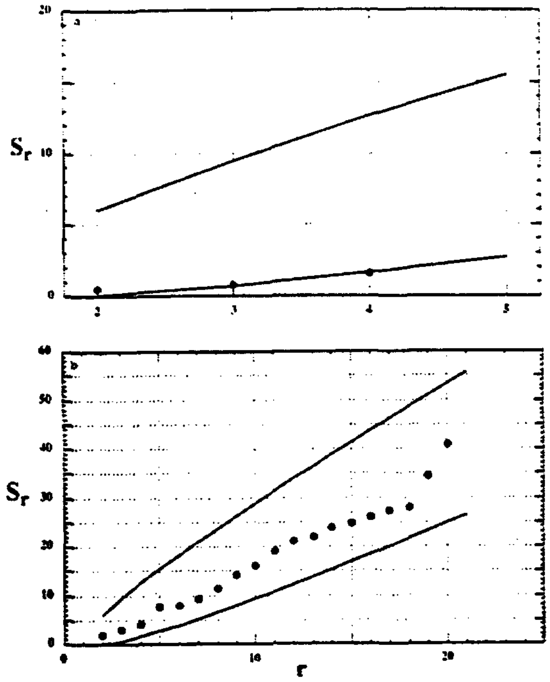

Figure 4. Control charts for Vesuvius based on 90% control limits (FE): (a) stage 1, (b) stage 2.

DOE-NRC Technical Exchange on Volcanism Studies

Comments by

C. -H. HO

Department of Mathematical Sciences

University of Nevada, Las Vegas

(this work is supported by the Nevada Nuclear Waste Project Office)

June 9, 1993

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$$P_{r_{dr}} = P_r(E_3 \text{ given } E_2, E_1)$$

$$\times P(E_2 \text{ given } E_1)$$

$$\times P(E_1)$$

where

WRONG!

E_1 denotes the recurrence rate of volcanic events, ... etc.

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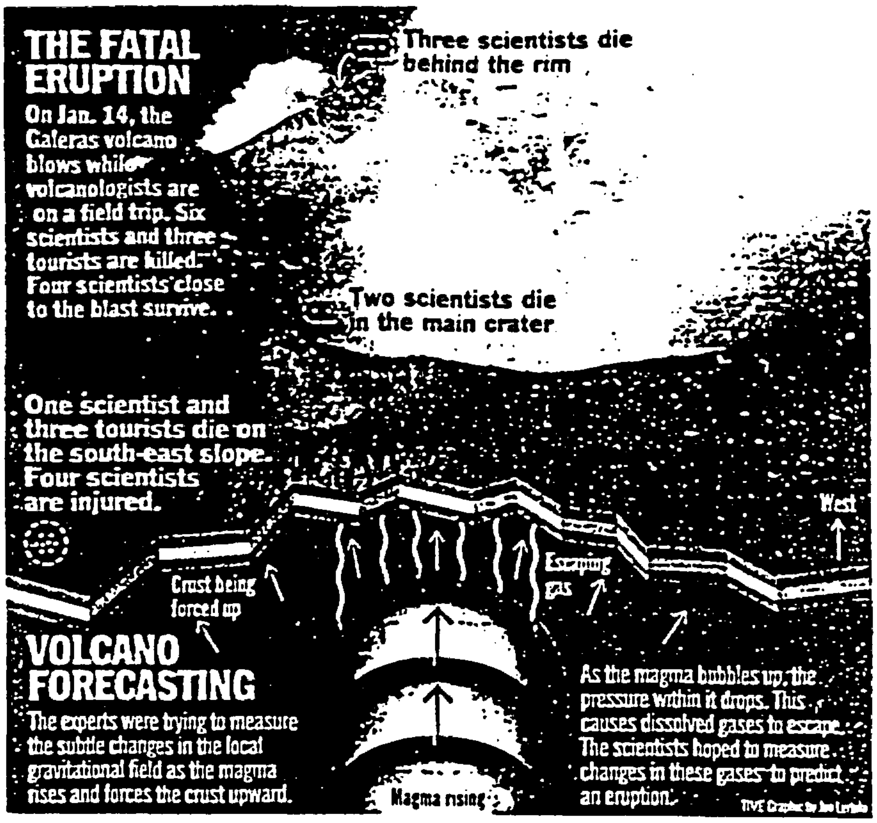
"RISK"

assessment of
Probability & CONSEQUENCES of
repository disruption

Hard!



THE FATAL ERUPTION

On Jan. 14, the Galeras volcano blows while volcanologists are on a field trip. Six scientists and three tourists are killed. Four scientists close to the blast survive.

Three scientists die behind the rim

Two scientists die in the main crater

One scientist and three tourists die on the south-east slope. Four scientists are injured.

VOLCANO FORECASTING

The experts were trying to measure the subtle changes in the local gravitational field as the magma rises and forces the crust upward.

As the magma bubbles up, the pressure within it drops. This causes dissolved gases to escape. The scientists hoped to measure changes in these gases to predict an eruption.

TIME Graphic by Joe Lertola

NATURE

DEADLY SCIENCE

A sudden and fatal eruption in Colombia shows again that volcanology is tragically imprecise

By LARA MARLOWE

WHAT LITTLE VOLCANOLOGISTS have learned over the centuries has come at a fearsome price. Beginning in A.D. 79, when the Roman scientist Pliny the Elder was killed while observing an eruption of Mount Vesuvius, volcanology has been one of the world's more dangerous fields of study. Over the past 11 years, sudden eruptions—including major blasts in Colombia, Mexico and the Philippines—have killed an estimated 26,000 people; since 1979 at least 12 scientists have perished while seeking to plumb the fiery mysteries.

Last month, to improve methods for predicting eruptions and thus save lives, [illegible] scientists from around the world gathered for a U.N.-sponsored conference in the southwestern Colombian city of Pasto. New techniques for detecting pre-eruption changes in the composition of vented gases had shown theoretical promise, and the scientists hoped to test them on Galeras, an active volcano several kilometers to the west that had not erupted since July 1992. Once again though, the insights of science were employed too late to be effective.

On the morning of Jan. 14, Stanley Williams, a U.S. volcanologist from Arizona State University, led a team of nine other scientists to the 4,170-m summit. Williams stayed on the rim and watched as two colleagues clambered down ropes toward the volcano's inner cone—Nestor Garcia, a Colombian, to set up a temperature probe; Igor Menyailov, a Russian, to sample gases coming out of vents. Williams and Menyailov, who had taught himself English by listening to Elvis Presley records, had been friends since they first met in 1982 on a volcano watch in Nicaragua. "Igor was excited because he was using a new device," Williams recalled last week from a hospital in Phoenix, Arizona. "He was smoking a cigarette, and he was all happy." Andrew McFarlane, of Florida International University, had just taken a snapshot of the two men when, without the slightest warning, the ground heaved and the mountain erupted.

"The volcano seemed to take a big breath, first sucking in air, then exploding," said a Colombian tourist who survived unhurt. Garcia and Menyailov died in an instant in the 600°C blast of toxic gases. On the western rim of the cone, British geologist Geoffrey Brown and two Colombian colleagues were also incinerated as gas and heat spurted upward.

"After seeing those people die," Williams recalled last week, "I just said 'Goddammit, I don't want to die,' and I started running as fast as I could." Scrambling down the slippery, ash-coated outer slope of the cone, he and three other scientists were bombarded with boulders the size of TV sets. "They split open when they hit the ground," said McFarlane. "Inside they were glowing red." One of the flying boulders crushed to death Colombian geochemist José Arles Zapata. Williams was felled as well, but managed to drag himself to partial shelter behind a huge rock.

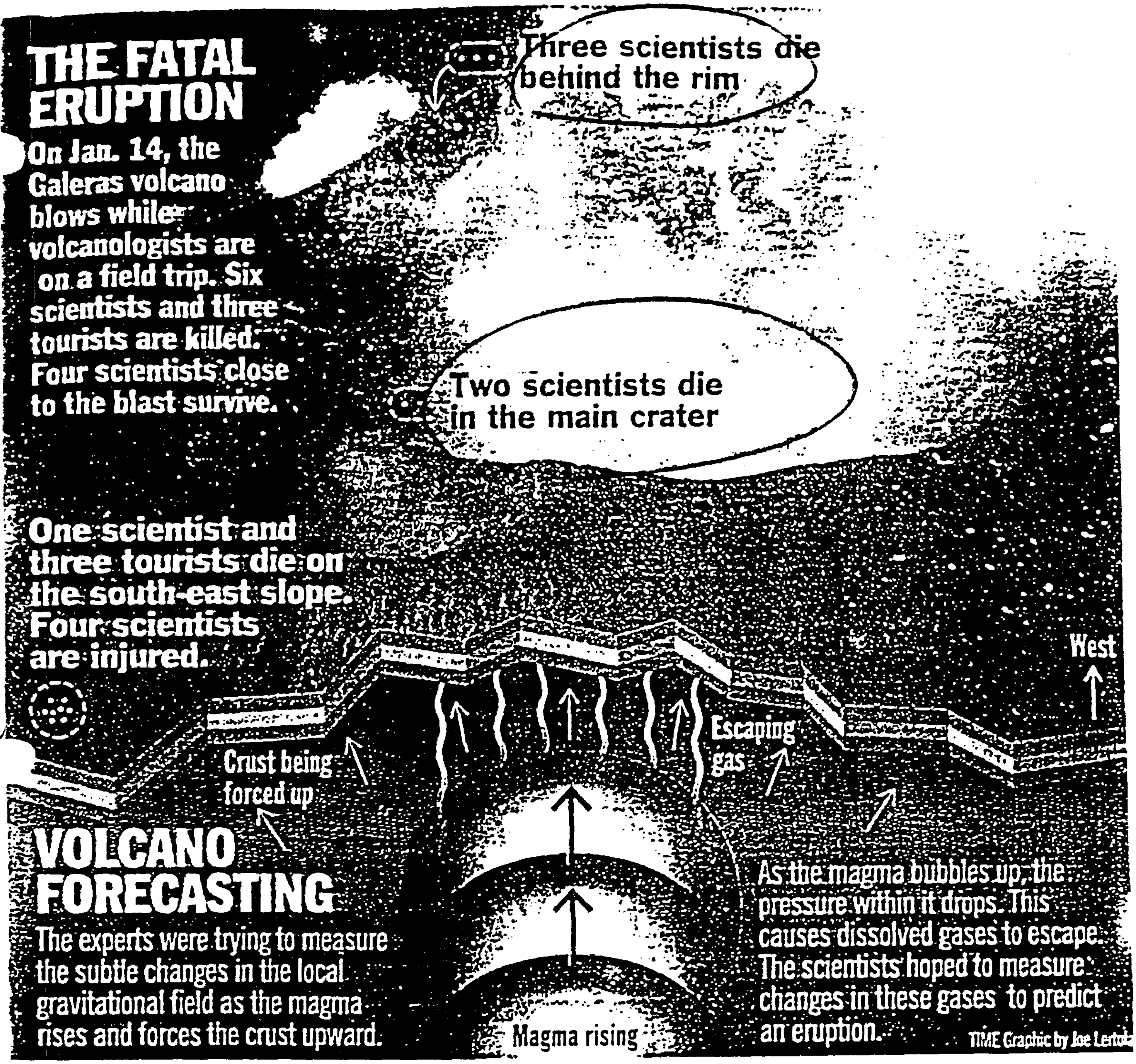
Williams and Mike Conway, from Michigan Technological University, said the thought of their wives and children made them determined to survive; McFarlane remembers wishing he had told his aging father that he loved him. "I was sure we were all going to die," he said. "The violence was shocking. Nature doesn't care—there was no mercy out there."

Stunned by a skull fracture, blinded by blood flowing down his forehead, his hands scorched, McFarlane at first tried to carry Williams, whose jaw and both legs were broken. "I was dazed from the impact and I was too weak to carry him, so I just kept running," said McFarlane. "I felt pretty guilty. I was very glad he made it." When rescuers finally reached the four survivors two hours later, they found Williams' backpack, altimeter and sunglasses melted and $6,000 in traveler's checks burned in his pocket. Somehow he was alive.

Conway was the only survivor able to walk away from Galeras; on his way out he passed the body of a dead tourist whose shirt was still on fire. The fourth survivor, Ecuadoran scientist Luis Lamarie, had to be carried out on a stretcher.

After learning of the deaths of their six colleagues and the three Colombian tourists, many of the volcanologists attending the Pasto conference quietly left. The few who remained for the final session completed proposals to pursue gravity and gas analysis forecasting. The deaths on the mountain also led them to call for more rigorous safety measures on volcanic sites—and to demand an end to tourism at Galeras. Visitors are no longer permitted to approach the volcano.

Williams is recovering. His jaw is wired shut, and doctors will graft bone from his pelvis to replace crushed leg bones. The explosion, he says, "shows how unpredictable these volcanos are, even for so-called experts like ourselves." He relives Galeras in nightmares, yet he feels driven to find more answers. He says he will resume his work. *—Reported by Patrick E. Cole/Phoenix and Tom Quinn/Bogota*



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TIME Graphic by Joe Lertola

NATURE

DEADLY SCIENCE

sudden and fatal eruption in Colombia shows again that volcanology is tragically imprecise

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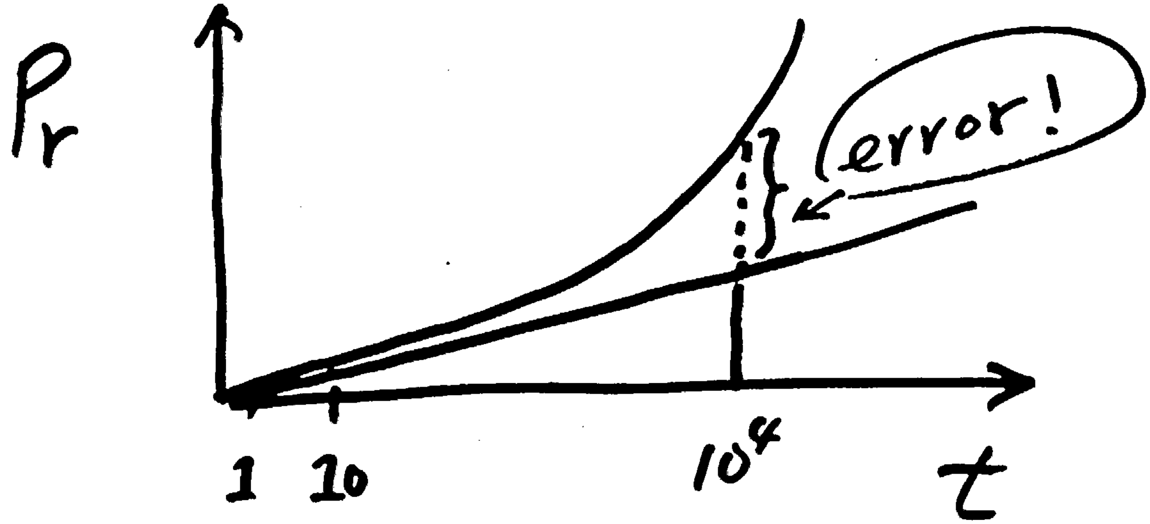
1. Can volcanic risk be expressed as an annual value? i.e. $10^{-8}\ \mathrm{yr}^{-1}$

2. $P_r(t) = \lambda t$, $e^{-\lambda t}$, $\theta e^{-\lambda t}$



No!

Misleading!

The **annual** disease-free survival rate after operation of lung cancer patients is xx percent.

*The estimated **annual** probability of future volcanic eruptions is* 5.5×10^{-6}.

The following statements, which reflect the projected time frame, are more informative.

The overall cumulative five-year disease-free survival rate is xx percent.

The estimated risk for an isolation time of 10^4 *years is about 5%, which increases to 42% if* 10^5 *years is the required isolation time.*

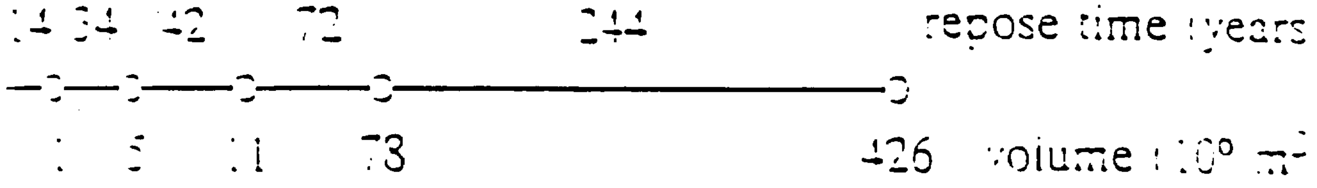
nonlinear

$42\% \neq 5\% \times 10^5$

What is
steady state eruptive behavior?

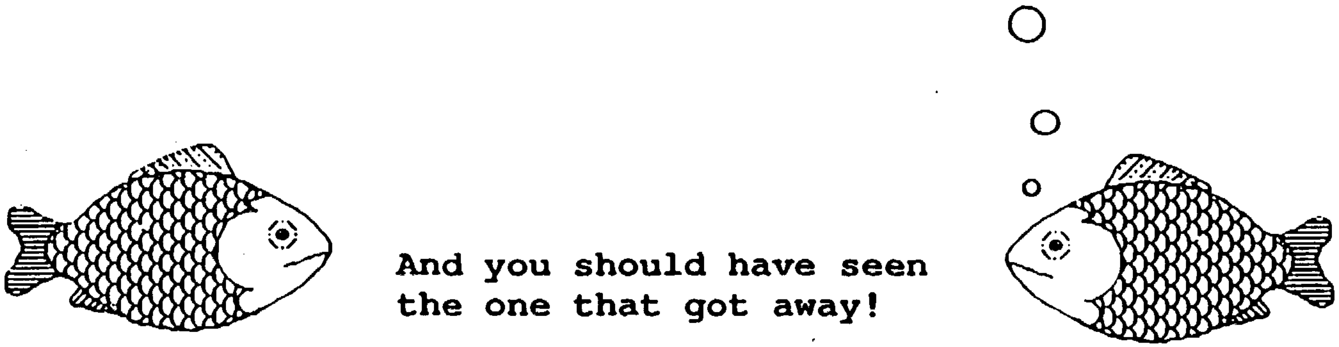
Time trend?

First, a simple Poisson model requires a constant rate of occurrence, which is not the same as magma production. Also, the trend of magma volume has not proved relevant to that of the frequency of the volcanic events (eruptions) at the NTS area. For example, the data set which has a waning time trend can also have an increasing trend in magma volume. The following data demonstrate this possibility:

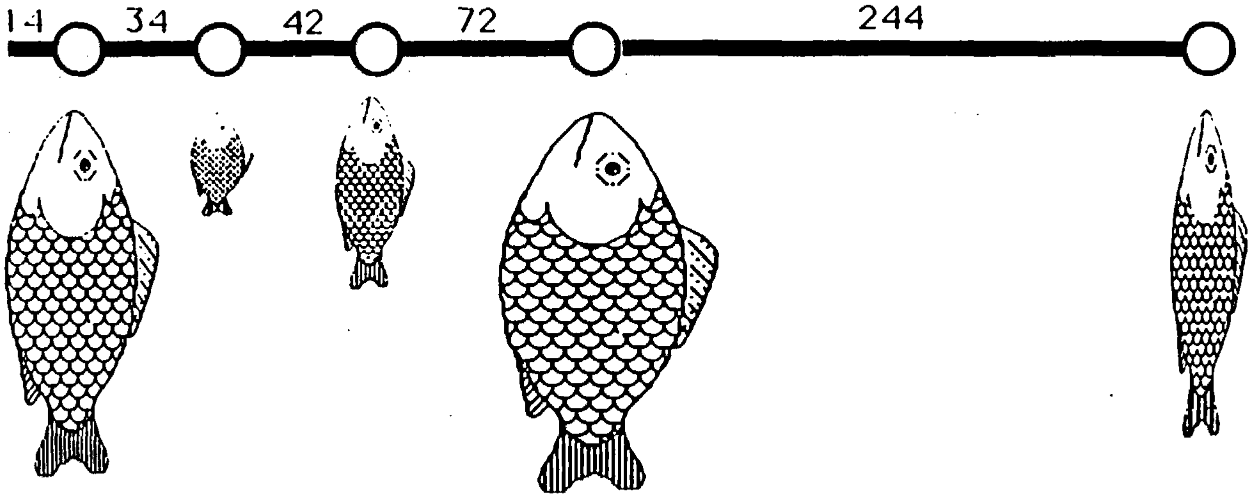


w.r.t. what??

Time trend in a "small data set."



And you should have seen the one that got away!



1. **GENERALIZE a constant λ with $\lambda(t)$, a function of time**

2. **Model $X(t)$ = number of events in $[0,t]$**

$X(t)$ follows a nonhomogeneous Poisson process (NHPP) with parameter $\mu(t)$

$$\mu(t) = \int_0^t \lambda(s)\,ds$$

(Parzen, 1962, p. 138)

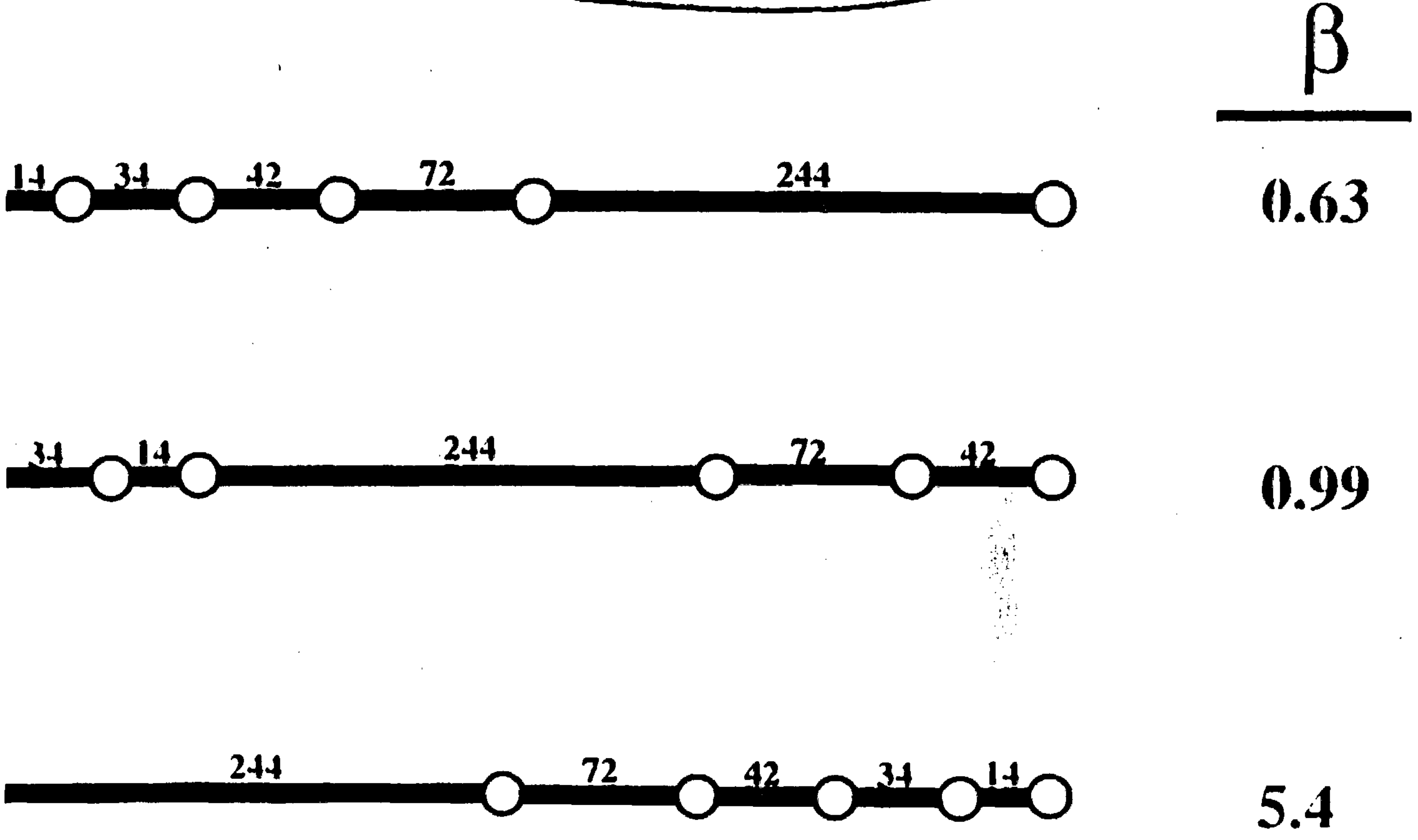
- **Choice of** $\lambda(t) = (\beta/\theta)\,(t/\theta)^{\beta-1}$
- **yields** $\mu(t) = (t/\theta)^{\beta}$
- **implies a Weibull** $(\theta\,,\,\beta)$

$$\beta \begin{cases} > 1 \ \textbf{increasing} \\ = 1 \ \textbf{simple Poisson} \\ < 1 \ \textbf{decreasing} \end{cases}$$

Weibull includes "a simple Poisson"

Weibull process modeling

= A car with auto-Transmission



A Simple Poisson process

= A car with only one gear.

REMARKS

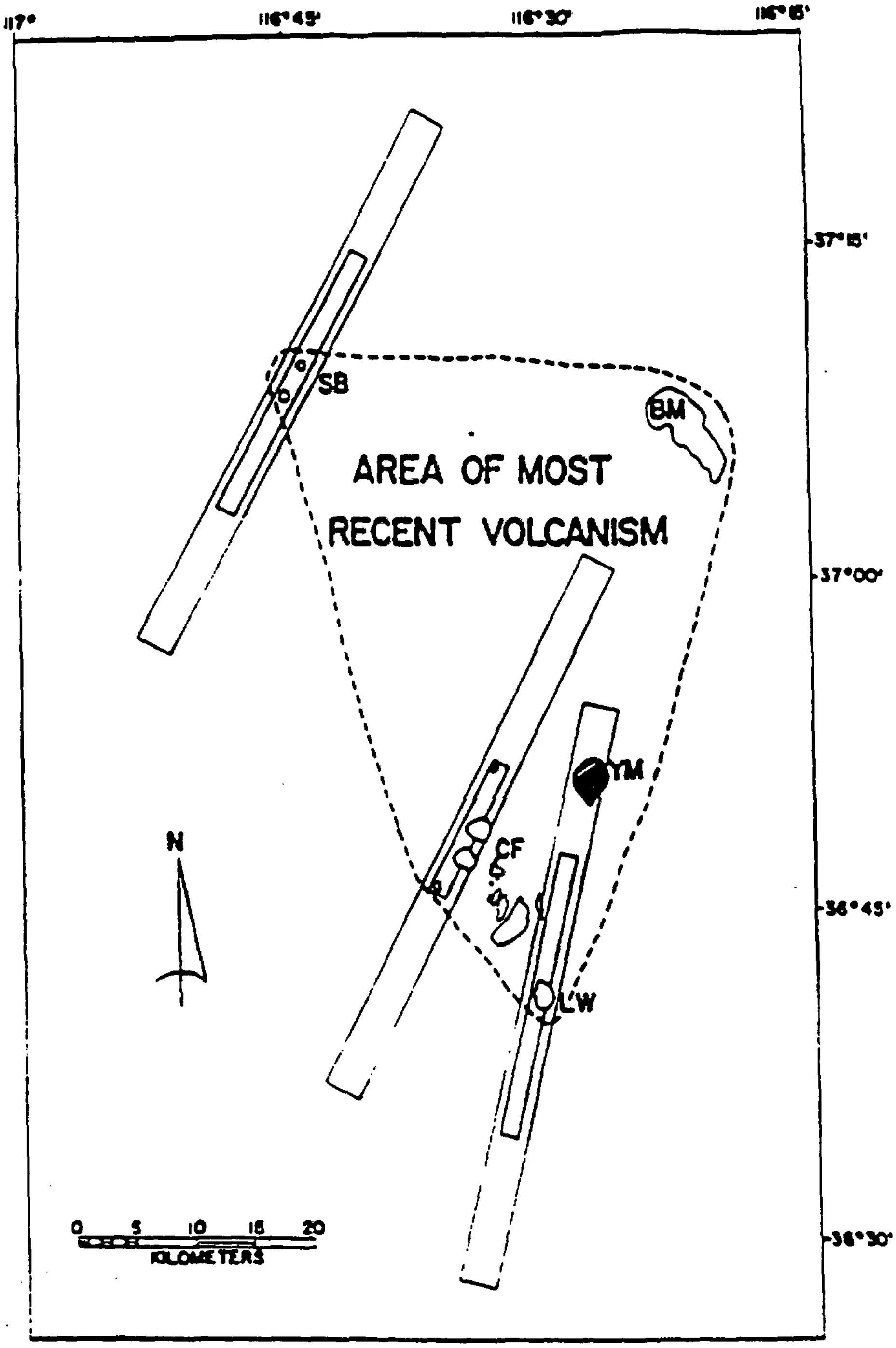
1. In this study, we restrict the risk to bull's-eyed volcanic events which result in the formation of volcanic cones and site disruption.

2. In so doing we neglect the potential impact of all other types of events such as a series of dikes, plugs, and sills, etc.

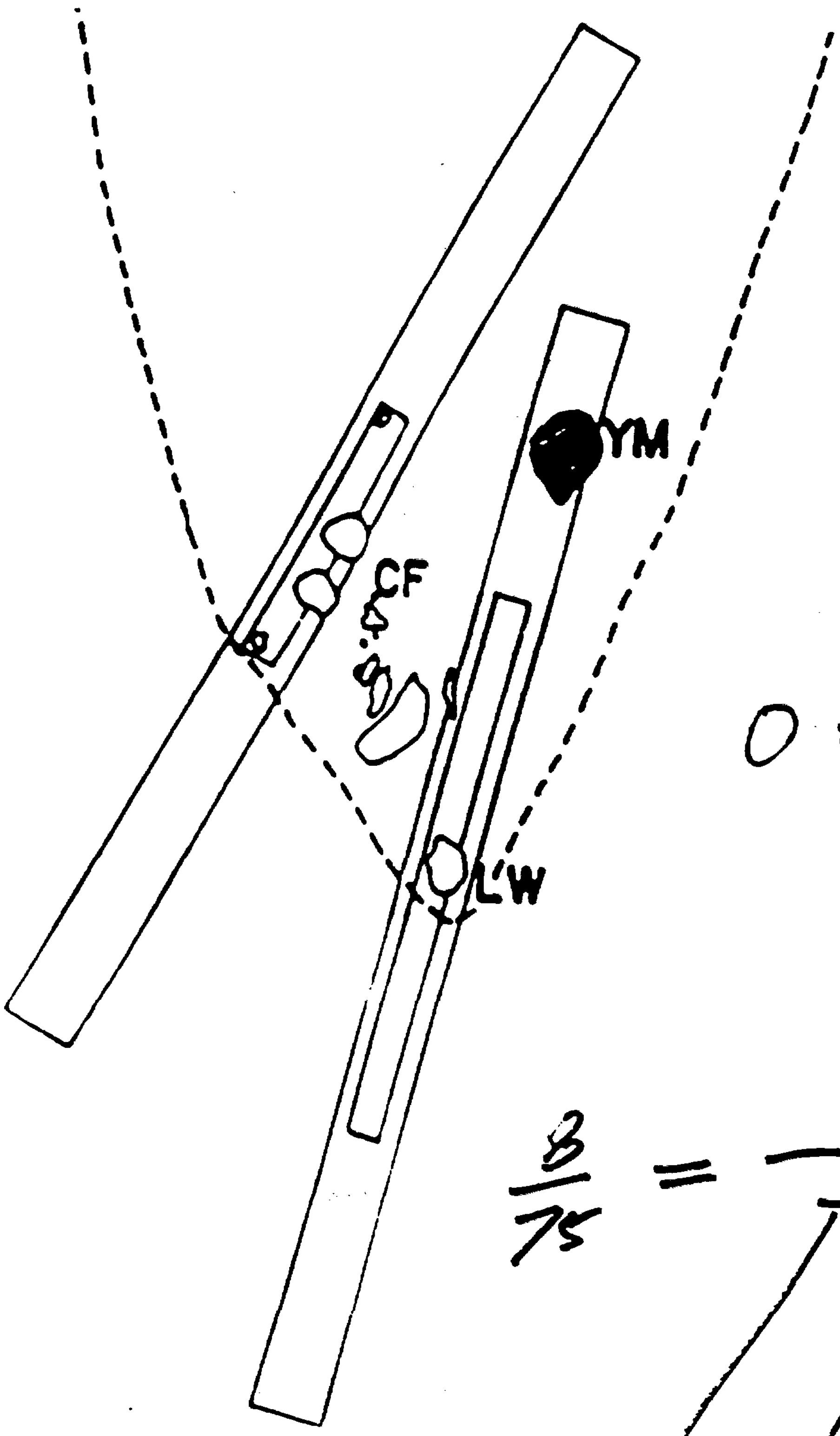
(What goes on under the surface?)

$$\text{Risk} = 1 - \int_p \exp\{-\lambda(t) p t_0\} \pi(p)\, dp$$

The technical machinery (Bayesian approach) involved in the risk calculation would support much more informative answers if the prior distribution $\pi(p)$ is adequately chosen.

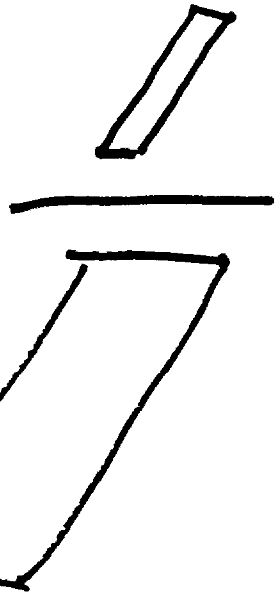


Map outlining the AMRV (dashed line) and high-risk zones (rectangles) in the Yucca Mountain (YM) area that include Lathrop Wells (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), volcanic centers within Crater Flat (CF). (Source: Smith et al., 1990a, fig. 7)



0 = lower limit

$\frac{8}{75}$ =



= upper limit

$\pi(p) \sim U(0, 8/75)$

We have

1. $A = 75\ \mathrm{km}^2$ (= half of the rectangle)

2. $a = 8\ \mathrm{km}^2$ (area of the repository,
Crowe et al, 1982)

3. $\pi(p) \sim U(0, 8/75)$, which assumes
8/75 as the upper limit for p

RESULT

A 90% confidence interval for the probability of site disruption for an isolation time of 10^4 years is

$$\left(1.0 \times 10^{-3}\,,\ 6.7 \times 10^{-3}\right)$$